Fall 2013-Exam 4: §8\&11-11/14/13-Write all responses on separate paper. Show your work for credit.

1. Convert the rectangular equation to polar coordinates and solve for $r$.
(a) $x^{2}+(y-4)^{2}=16$
(b) $\left(x^{2}+y^{2}+y\right)^{2}=4\left(x^{2}+y^{2}\right)$
2. Convert the polar equation to rectangular coordinates and solve for $y$.
(a) $r=\frac{1}{\sin \theta+\cos \theta}$
(b) $r=\sec \theta(\tan \theta-1)$
3. Consider the polar function $r=\frac{2}{1-\sin \theta}$
(a) Test the function for symmetry. What do you find?
(b) Write the function as a conic section in standard rectangular form.
(c) Complete the table below for $r, x$, and $y$ for the given $\theta$

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{5 \pi}{6}$ | $\pi$ | $\frac{7 \pi}{6}$ | $\frac{3 \pi}{2}$ | $\frac{11 \pi}{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ |  |  |  |  |  |  |  |
| $x$ |  |  |  |  |  |  |  |
| $y$ |  |  |  |  |  |  |  |

(d) construct a graph for the function.

4. Find all solutions to each equation, including the complex solutions. Hint: first convert the number to polar form and use DeMovire's theorem.
(a) $x^{5}=-1$
(b) $x^{6}=8+15 i$
5. Consider the ellipse described by $\frac{(x-4)^{2}}{25}+\frac{y^{2}}{9}=1$
(a) Find the center, $x$-intercepts, $y$-intercepts and the coordinates of the foci.
(b) Sketch a graph showing these features.
(c) What is the eccentricity, $e=\frac{c}{a}$ ?
(d) What is the polar form? Hint: it's in the $r=\frac{e d}{1-e \cos \theta}$ form
6. Consider the hyperbola describe described by $r=\frac{10}{2-3 \sin \theta}$
(a) Find the eccentricity.
(b) Complete the table:

| $\theta$ | 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3 \pi}{2}$ | $\arcsin \left(-\frac{12}{13}\right)$ | $\pi-\arcsin \left(-\frac{12}{13}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ |  |  |  |  |  |  |
| $x$ |  |  |  |  |  |  |
| $y$ |  |  |  |  |  |  |

(c) Given that the vertices of the hyperbola are the $y$-intercepts what are the coordinates of the center?
(d) (4 points) Sketch a graph (see attached graph paper).
(e) (4 points) What is the rectangular form?
7. Find parametric equations for each given conic.
(a) $\frac{x^{2}}{4}+\frac{(y-1)^{2}}{9}=1$
(b) $(x-1)^{2}-y^{2}=1$
(c) $4(y-1)=(x-2)^{2}$
8. Make a table of values and sketch a graph for the given parametric equations.

$$
\begin{align*}
x & =\cos (t)  \tag{1}\\
y & =\sin ^{2}(t) \tag{2}
\end{align*}
$$



