Math 12 **Name (Print):** ____________________

Fall 2016 - Exam 3: Chapter 7 - 11/10/16 - Write all responses on separate paper. Show your work for credit. Do not use an electronic calculator.

---

1. (10 points) Simplify the expression: $\dfrac{2+\tan^2(x)}{\sec^2 x}$

2. Prove the identity:
   (a) $(\sin x + \cos x)^2 - 1 = \sin(2x)$
   (b) $\dfrac{\sin\theta}{1-\cos\theta} - \cot\theta = \csc\theta$

3. Prove that $\cos(\alpha+\beta)\cos(\beta) + \sin(\alpha+\beta)\sin(\beta) = \cos\alpha$.

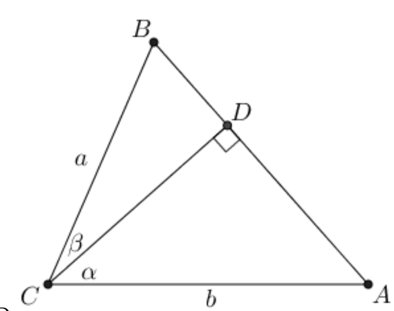
4. Find values of C and ϕ so that the given sum is equal to $C\sin(\omega t + \phi)$:
   (a) $\sqrt{2}\cos(5x) + 2\sin(5x)$
   (b) $\sqrt{3}\sin(\pi x) + 2\cos(\pi x)$

5. Show that $\dfrac{\sqrt{2}+\sqrt{6}}{2} = \dfrac{\sqrt{2+\sqrt{3}}}{2}$ by considering that $\sin\left(\dfrac{5\pi}{12}\right) = \sin\left(\dfrac{\pi}{6}+\dfrac{\pi}{4}\right) = \sin\left(\dfrac{5\pi/6}{2}\right)$

6. Use the addition identity for $\tan(x)$ to write $\tan(3x)$ in terms of only $\tan(x)$. Simplify. Verify that you formula is correct at $x = \dfrac{\pi}{4}$

7. (a) Prove that $\sin(x) + \sin(y) = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$.
   (b) Prove that $\cos(x) + \cos(y) = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$.
   (c) Prove that $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\dfrac{x+y}{2}\right)$

8. Consider the diagram at right which we'll use to prove the addition identity for $\sin(\alpha+\beta)$.
   (a) Express $\overline{AD}$ in terms of b and α.
   (b) Express $\overline{BD}$ in terms of a and β.
   (c) Express $\overline{CD}$ in two ways:
      - in terms of b and α.
      - in terms of a and β.
   (d) Express the area of $\triangle ABC$ in terms of α, β, a and b.
   (e) Use the fact that the area of $\triangle ABC$ is the sum of areas of $\triangle CBD$ and $\triangle ACD$ to prove the addition identity.



9. Solve the equation.
   (a) $(2\cos(3\theta) - 1)(2\cos(3\theta) + 1) = 1$
   (b) $\cos(3\theta) - \cos(7\theta) = 0$