1. Solve each equation for x. You won't need a calculator.

   (a) $2 = \log_5(x^2 + 3x - 3)$ Sol'n: $x^2 + 3x - 3 = 25 \Leftrightarrow x^2 + 3x - 28 = 0 \Leftrightarrow (x-4)(x+7) = 0 \Leftrightarrow$ $\boxed{x = 4 \text{ or } x = -7}$.

   (b) $4^{-0.5x} = \dfrac{1}{8}$

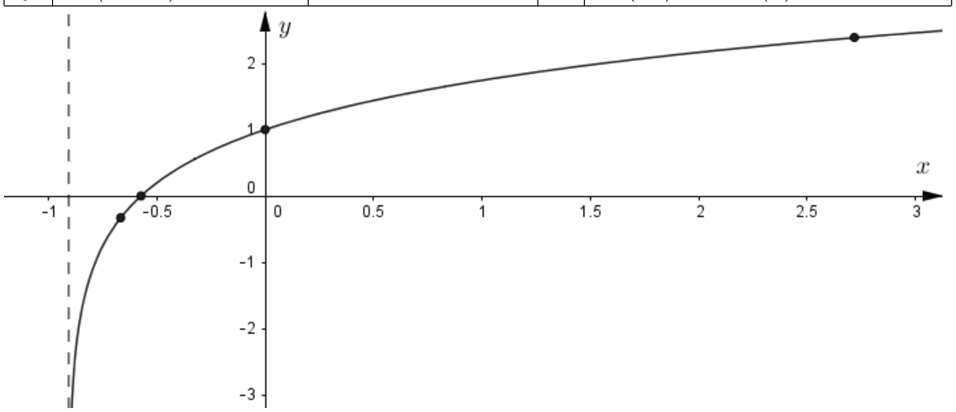
   Sol'n: $\dfrac{1}{2^x} = \dfrac{1}{8} \Leftrightarrow \boxed{= 3}$

2. For each function, (1) find the vertical asymptote(s), (2) find the x and $y-$intercepts, (3) construct a table of values including two other points and (4) construct a graph showing these features:

   (a) $f(t) = \ln(3t + e)$

   Sol'n: Require that $3t + e > 0 \Leftrightarrow t > \dfrac{-e}{3}$ Thus the vertical asymptote is $x = \dfrac{-e}{3}$. The intercepts are $(0, 1)$ and $\left(\dfrac{1-e}{3}, 0\right)$

x	$\dfrac{-2}{3}$	$\dfrac{1-e}{3} \approx -0.573$	0	e
y	$\ln(e-2) \approx -0.33$	0	1	$\ln(4e) = 2\ln(2) + 1 \approx 2.39$

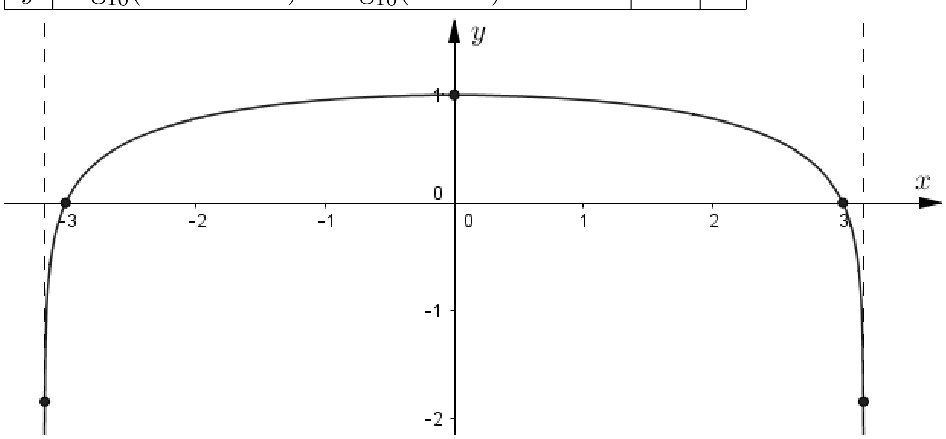


   (b) $g(x) = \log_{10}(10 - x^2)$

   Sol'n: Require that $10 - x^2 > 0 \Leftrightarrow x^2 < 10 \Leftrightarrow -\sqrt{10} < x < \sqrt{10}$ Thus the vertical asymptotes are $x = -\sqrt{10}$ and $x = \sqrt{10}$

   $f(0) = \log_{10}(10) = 1$ and $f(x) = 0 \Leftrightarrow 10 - x^2 = 1 \Leftrightarrow x = \pm 3$

x	± 3.16	± 3	0
y	$\log_{10}(10 - 9.9856) = \log_{10}(0.0144) \approx -1.84$	0	1



3. Solve the equation for x in an exact, simplified form, approximate to the nearest ten thousandth.
   (a) $2^{10} = 100^x$
   Sol'n: $2^{10} = 10^{2x} \Leftrightarrow 2x = \log_{10} 2^{10} \Leftrightarrow 2x = 10\log_{10}(2) \Leftrightarrow x = 5\log_{10}(2) \approx 5 \cdot 0.301029 \approx 1.5051$
   (b) $\left(1+\frac{1}{8}\right)^3 = e^{2x}$
   Sol'n: $2x = 3\ln\left(\frac{9}{8}\right) = \ln(9) - \ln(8) = 2\ln 3 - 3\ln 2 \approx 2(1.09861) + 3(0.69315) \approx 0.1767$

4. Use the properties of logarithms to solve each equation
   (a) $\log_5(3x-2) + \log_5(x-8) = 2$ Sol'n:$\log_5(3x-2) + \log_5(x-8) = 2 \Rightarrow \log_5(3x-2)(x-8) = 2 \Leftrightarrow (3x-2)(x-8) = 25 \Leftrightarrow 3x^2 - 26x - 9 = 0 \Leftrightarrow (3x+1)(x-9) = 0 \Leftrightarrow x = -\frac{1}{3}$ or $\boxed{x = 9}$ (only the latter is a valid solution.)
   (b) $\log_2(x^2+1) = 1 + \log_2(x+2)$ Sol'n: $\log_2\left(\frac{x^2+1}{x+2}\right) = 1 \Leftrightarrow \frac{x^2+1}{x+2} \Longleftrightarrow x^2+1 = 2x+4 \Leftrightarrow x^2 - 2x - 3 = (x-3)(x+1) = 0 \Leftrightarrow \boxed{x = -1 \text{ or } x = 3}$

5. Suppose that $1,000 is invested in a savings account paying 3.2% interest per year..
   (a) Write the formula for the amount in the account after t years if interest is compounded monthly.
   Sol'n: $A(t) = 1000\left(1 + \frac{0.032}{12}\right)^{12t} = 1000(1.002\overline{6})^{12t} \approx 1000(1.03247353)^t$
   (b) How much more would the value of the investment increase if it was compounded continuously?
   $1000e^{0.032t} - 1000(1.03247353)^t$. After a year, $\approx 1032.5175 - 1032.47353 \approx 0.044$, about 4 cents.

6. The Richter magnitude of an earthquake is defined to be $M = \log\frac{I}{S}$ where I is the intensity of the earthquake and S is the intensity of a "standard" earthquake. An earthquake measuring 9.0 on the Richter scale struck Japan in March 2011, causing extensive damage. How many times more intense was the Japanese earth-quake than a minor earthquake measuring 4.3 on the Richter scale?
   Hint: If I_0 is the intensity of the Japanes earthquake and I_1 is the intensity of the Salton Sea quake, we want to compute $\frac{I_0}{I_1}$.
   Sol'n: We have $9 = \log\frac{I_0}{S} \Leftrightarrow I_0 = S \cdot 10^9$ and $4.3 = \log\frac{I_1}{S} \Leftrightarrow I_1 = S \cdot 10^{4.3}$ so $\frac{I_0}{I_1} = \frac{S \cdot 10^9}{S \cdot 10^{4.3}} = 10^{4.7} = 10^{0.7} \times 10^4 \approx 50000$ times more intense.

7. Postassium-40 (^{40}K) is a radioactive isotope of potassium which has a very long half-life of 1.251×10^9 years. At time $t = 0$ a heavy canister contains 3 grams of Potassium-40.
   (a) Find a function $m(t) = m_0 2^{-t/h}$ that models the amount of ^{40}K left in the canister after t years.
   Sol'n: $m(t) = 3 \cdot 2^{-t/(1.251 \cdot 10^9)} \approx 3 \cdot 2^{-7.994 \cdot 10^{-10} t}$
   (b) Find a function $m(t) = m_0 e^{-rt}$ that models the amount of ^{40}K remaining after t seconds.
   Sol'n: We seek r so that $3 \cdot 2^{-t/(1.251 \cdot 10^9)} = 3 \cdot e^{-rt} \Leftrightarrow r = \frac{\ln 2}{1.251 \cdot 10^9} \Rightarrow r \approx 5.5407 \times 10^{-10}$ so $\boxed{m(t) \approx 3e^{-5.5407 \cdot 10^{-10} t}}$ If you want this in seconds(?!) then use the fact that 1 year $\approx 365.2422\texttt{days} \times \frac{24\texttt{hrs}}{\texttt{day}} \times \frac{3600\texttt{sec}}{\texttt{hr}} = 31556926.08$ seconds and so $m(t) \approx 3e^{-5.5407 \cdot 10^{-10}/31556926.08t} \approx 3e^{-1.756 \cdot 10^{-17} t}$
   (c) How much ^{40}K remains after 1 billion years?
   Sol'n: $m(10^9) = 3 \cdot 2^{-1/1.251} \approx 1.72$ grams.
   (d) After how long will the amount of ^{40}K be reduced to 1 mg$= 10^{-6}$ grams?
   Sol'n: Solve $m(t) = 3 \cdot 2^{-t/(1.251 \cdot 10^9)} = 10^{-6} \Leftrightarrow \frac{-t}{1.251 \cdot 10^9} = \log_2\left(\frac{10^{-6}}{3}\right)$
   $\Leftrightarrow t = -1.251 \cdot 10^9(-6 - \log_2(3)) = 1.251 \cdot 10^9(6 + \frac{\ln 3}{\ln 2} \approx 9.49$ billion years. Don't wait up.