Math 5 - Fall '16 - Test 1 **Name (Print):** ______________________

Write all responses on separate paper.
Show work for credit. No calculator/notes.

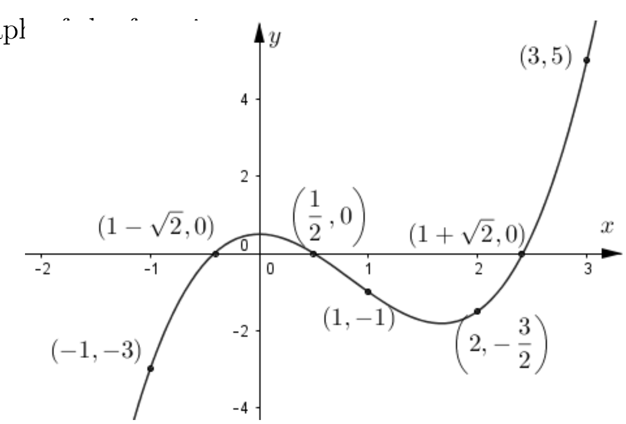
1. A bowling ball thrown by a bowler follows a parabolic trajectory given by the graph of the equation $h(t) = 4 - 4\left(t - \frac{1}{2}\right)^2$, where t is the time since ball was thrown and $h(t)$ is the height of the ball above the ground at time t.
   (a) What is the maximum height the bowling ball reaches?
   Sol'n: The formula for the parabola is given in vertex form, so you can just read off the coordinates of the maximum: $(h, k) = \left(\frac{1}{2}, 4\right)$ shows that the bowling ball reaches its maximum height of 4 ft. Also, while the question doesn't ask for this, the maximum height of 4 feet happens at time $t = \frac{1}{2}$.
   (b) When does the bowling ball hit the ground?
   Sol'n: Solve $h(t) = 0 \Leftrightarrow 4 = 4\left(t - \frac{1}{2}\right)^2 = 4 \Leftrightarrow t - \frac{1}{2} = \pm\sqrt{1} \Leftrightarrow t = \frac{1}{2} \pm 1$. Since t must be positive, we take $t = \frac{3}{2} = 1.5$ seconds.

2. Consider the polynomial function $f(x) = x^3 - \frac{5}{2}x^2 + \frac{1}{2}$
   (a) Explain why this function does not satisfy the condition for the theorem on rational zeros. Write a function with the same zeros that does satisfy the condition.
   Sol'n: The condition for applying the theorem on rational roots is that the coefficients of the polynomial are integer, but $\frac{5}{2}$ is not an integer; that is, $\boxed{\frac{5}{2} \notin \mathbb{Z}}$.
   (b) List all the possible rational zeros of the function.
   Sol'n: Observe that the zeros of $2f(x) = 2x^3 - 5x^2 + 1$ are the same as the zeros of f so we can apply the theorem on rational roots to $2f(x)$ to find all possible roots of f : $\boxed{x \in \{1, \frac{1}{2}\}}$.
   (c) Write a complete factorization for the function.
   First we need to find a rational zero.

x				$2y$
0	2	−5	0	1
1	2	−3	−3	−2
$\frac{1}{2}$	2	−4	−2	0
2	2	−1	−2	−3
3	2	1	3	10
−1	2	−7	7	−6

We get a lot of information from the synthetic division table at left. Dividing by $x - 1$ produces a remainder of -2, so we know $2f(1) = -2$ and further, since $2f(0) = 1$ is positive, the function changes sign in $(0, 1)$ and thus by the Intermediate Value Theorem, it must have a zero in $(0, 1)$. So we divide by $x - \frac{1}{2}$ and find that $\frac{1}{2}$ is a zero, so $2f(x) = (2x - 1)(x^2 - 2x - 1)$. Zeros of $x^2 - 2x - 1 = (x - 1)^2 - 2$ are $x = 1 \pm \sqrt{2} \approx 2.4$ and -0.4. So a complete factorization is $\boxed{f(x) = \frac{1}{2}(2x - 1)(x - 1 + \sqrt{2})(x - 1 - \sqrt{2})}$.

   (d) construct a careful grapl



3. Consider the polynomial function $p(x) = 3x^4 + 10x^3 + 4x^2 - 5x - 2$.

   (a) What does Descartes' rule of signs say about the number of positive and negative zeros of p?
   Soln: There is exactly one positive zero and 3 or 1 negative zeros.

   (b) Use the Remainder Theorem to evaluate $p(-4)$ and find $q(x)$ so that $p(x) = (x+4)q(x)+p(-4)$.

x					y
0	3	10	4	-5	-2
-4	3	-2	12	-53	210

Thus $p(x) = (x+4)(3x^3 - 2x^2 + 12x - 53) + 210$

   (c) Find $p(1)$ and explain why there must be a zero in the interval $(0, 1)$
   Soln: As the synthetic division tableau below shows, $p(0) = -2$ and $p(1) = 10$, so, by the Intermediate Value Theorem, there is some $r \in (0, 1)$ such that $p(r) = 0$.

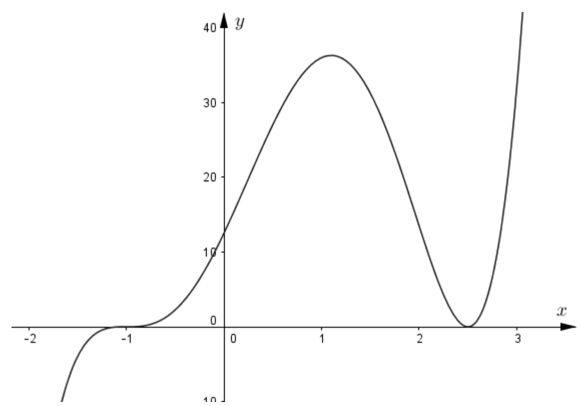
   (d) List all the possible rational zeros of $p(x)$, according the theorem on rational zeros.
   Soln: $\pm\{1, 2, \frac{1}{3}, \frac{2}{3}\}$.

   (e) Find all the zeros of $p(x)$.

x					y
0	3	10	4	-5	-2
1	3	13	17	12	10
$\frac{1}{3}$	3	11	$\frac{23}{3}$	$-\frac{22}{9}$	$-\frac{76}{27}$
$\frac{2}{3}$	3	12	12	3	0
$\frac{2}{3}$	3	14	$\frac{64}{3}$	$\frac{155}{9}$	
-1	3	9	3	0	

The synthetic division tableau, together with the Remainder Theorem shows that $f(0) = -2$ and $f(1) = 10$ so that, by the Intermediate Value Theorem, there is a zero in $(0, 1)$. The rational number, $\frac{1}{3}$ is a contender, but we find that $f\left(\frac{1}{3}\right) = -\frac{76}{27}$, so there is a zero in $\left(\frac{1}{3}, 1\right)$. The rational number $\frac{2}{3}$ is a contender and we find that $f(x) = \left(x - \frac{2}{3}\right)(3x^3 + 12x^2 + 12x + 3)$ and since $\frac{2}{3}$ is an upper bound on the zeros, we look at the negatives where we quickly see that $f(-1) = 0$ and so $f(x) = (3x - 2)(x + 1)(x^2 + 3x + 1)$ and the zeros are
$$\boxed{\frac{2}{3}, -1, -\frac{3}{2} + \frac{\sqrt{5}}{2}, -\frac{3}{2} - \frac{\sqrt{5}}{2}}$$

4. Write a formula for the polynomial function of degree 5 whose graph is shown:
   Soln: A root of multiplicity 2 at $x = \dfrac{5}{2}$ and a root of multiplicity 3 at $x = -1$. There are 5 factors: $p(x) = a(x + 1)^3(2x - 5)^2$. Now we can determine the scaling factor, a, by requiring that $p(0) = 12.5 \Leftrightarrow 25a = 12.5 \Leftrightarrow a = \dfrac{1}{2}$. Thus $\boxed{p(x) = \frac{1}{2}(x + 1)^3(2x - 5)^2}$.



5. Consider the rational function, $r(x) = \dfrac{x^3 - 2x^2 - 3x}{x^2 - 4}$

   (a) What are the intercepts?
   Soln: $r(x) = \dfrac{x(x - 3)(x + 1)}{(x - 2)(x + 2)}$ so the intercepts are $(-1, 0), (0, 0)$, and $(3, 0)$.

   (b) What asymptotes does it have?
   Soln: There are vertical asymptotes along $x = -2$ and $x = 2$ and since the degree of the numerator is one more thant the degree of the denominator, there's a slant asymptote. To find

that, you need to do long division:

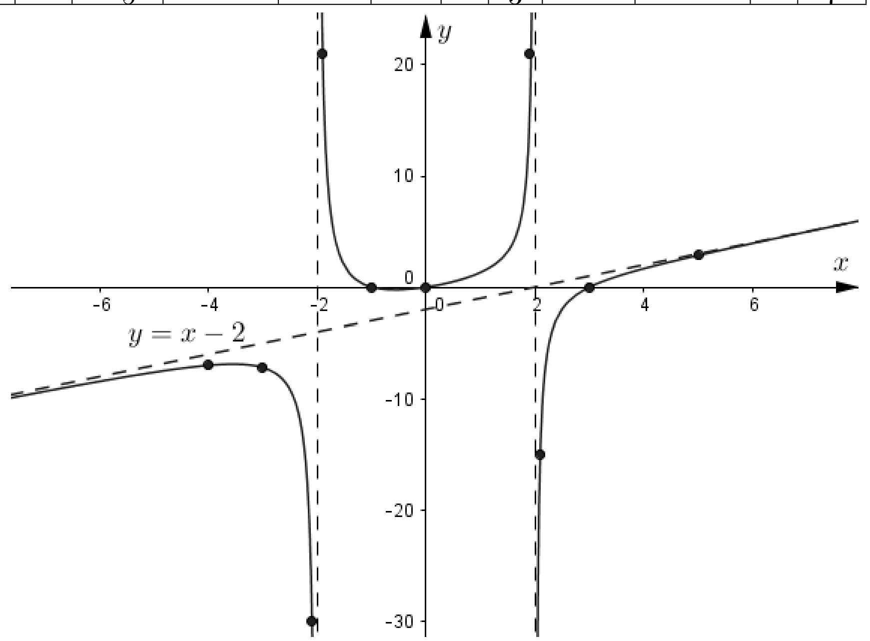
$$\begin{array}{r} x-2 \\ x^2-4 \overline{) \; x^3-2x^2-3x+0} \\ -x^3 \qquad\quad +4x \quad \\ \hline -2x^2 \; +x+0 \\ 2x^2 \qquad -8 \\ \hline x-8 \end{array}$$

So $r(x) = x-2+\dfrac{x-8}{x^2-4}$ and when $|x| >> 0$ (x is far from zero), $\boxed{y \approx x-2}$, the slant asymptote.

(c) Construct a table of values and a graph for the function.

Soln:

x	-4	-3	-2.1	-1.9	-1	0	1	1.9	2.1	3	5
$r(x)$	-7	$-\dfrac{36}{5}$	≈ -30	≈ 21	0	0	$\dfrac{4}{3}$	≈ 15	≈ -15	0	$\dfrac{20}{7}$



6. Solve the inequality $x \geq \dfrac{16-x}{2x+3}$

Soln: $\Leftrightarrow \dfrac{x(2x+3)+x-16}{2x+3} \geq 0 \Leftrightarrow \dfrac{2x^2+4x-16}{2x+3} = \dfrac{2(x+4)(x-2)}{2x+3} \geq 0 \Leftrightarrow x \in [-4,-3/2) \cup [2,\infty)$

7. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{2x^3+7x^2+4x-4}}$

Soln: Domain$= \{x | 2x^3+7x^2+4x-4 > 0\}$. To solve the inequality, we need to factor the cubic, so it'd be real nice to have a rational zero, which must be in $\pm\{1,2,4,\frac{1}{2}\}$. Hunt and peck with synthetic division:

x				y
0	2	7	4	-4
1	2	9	13	9
$\frac{1}{2}$	2	8	8	0

We can factor the cubic!:
$(2x-1)(x^2+4x+4) = (2x-1)(x+2)^2 > 0 \Leftrightarrow$. Since $-2 < \frac{1}{2}$, the domain is $\boxed{x \in (\frac{1}{2}, \infty)}$