## Math 12 - Precalculus <br> Final Exam

Name (Print):

Write your responses on the pages provided or attach additional pages, as needed. No notes are allowed. A scientific calculator is allowed, but not a graphing calculator. You can specify here which one (1) problem you'd prefer to exclude:

1. Find all zeros for each polynomial function.
(a) $f(x)=2 x^{3}+x^{2}-3 x+1$.
(b) $p(x)=x^{5}-3 x^{3}+x=x\left(x^{2}+x-1\right)\left(x^{2}-x-1\right)$.
2. We seek a formula for the polynomial function with integer coefficients whose graph is shown.
(a) What does the $y$-axis symmetry tell you about the polynomial?
(b) What can you deduce about the polynomial from its behavior at $(0,0)$ ?

(c) What does the root at $(1.5,0)$ tell you about the polynomial (given there are integer coefficients.)?
(d) Find an expression for the polynomial.

Hint: it passes through $(1,25)$.
3. Consider the rational function $R(x)=\frac{4 x^{3}-9 x}{x^{3}-1}$
(a) What are the $x$-intercepts of the function?
(b) What is the $y$-intercept?
(c) What vertical asymptote(s) are there, if any?
(d) What is the horizontal asymptote?
(e) Complete the table of values (approximate, as appropriate) and sketch a graph.

| $x$ | -10 | -2 | -1.5 | -1 | 0 | 0.5 | 1.1 | 2 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |  |  |  |  |


4. Solve each equation.
(a) $\log _{2}\left(x^{2}-32\right)-\log _{2}(x+8)=1$
(b) $4=\frac{10}{1+4 e^{-0.8 t}}$

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5. The half-life of polonium-210 is 138 days. Suppose we have a $100-\mathrm{g}$ sample.
(a) Find a function $m(t)=m_{0} 2^{-t / h}$ that models the mass remaining after t days.
(b) Find a function $m(t)=m_{0} e^{-r t}$ that models the mass remaining after t days.
(c) How much of the sample will remain after 400 days?
(d) After how many days will only 20 g of the sample remain?
6. For the angles $\alpha=\arctan (3 / 4), \beta=\arctan (\sqrt{3})$ simplify each of the following.
(a) $\sin (\alpha+\beta)$.
(b) $\cos (2 \alpha+\beta)$.
7. Find all solutions to each equation.
(a) $8 \sin ^{3}(x)-4 \sin ^{2}(x)-6 \sin (x)+3=0$ Hint: factor by grouping.
(b) $\sec \theta+\tan \theta=\frac{5}{3}$
8. Complete the table of values and plot the polar function. $r=\sin \theta+\cos \theta$

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2 \pi}{3}$ | $\frac{3 \pi}{4}$ | $\frac{5 \pi}{6}$ | $\pi$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ |  |  |  |  |  |  |  |  |  |



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9. Consider the ellipse whose equation is $\frac{x^{2}}{9}+\frac{(y-2)^{2}}{16}=1$
(a) Find the coordinates of center.
(b) Find the $x$-intercepts of the ellipse.
(c) Find the coordinates of the two foci.
(d) Write parametric equations for the ellipse.
(e) Sketch a graph for the ellipse
10. Consider the parametric equations

$$
\begin{aligned}
& x=2 \tan (t) \\
& y=3+4 \sec (t)
\end{aligned}
$$

(a) Eliminate the parameter to find an equation relating $x$ and $y$ directly. Hint: $\sec ^{2}(t)-\tan ^{2}(t)=1$.
(b) Tabulate values for $t, x$ and $y$ and use these to sketch a graph for the relation.

