Math 12 - Precalculus Final Exam Solutions - Fall 2016

1. Find all zeros for each polynomial function.

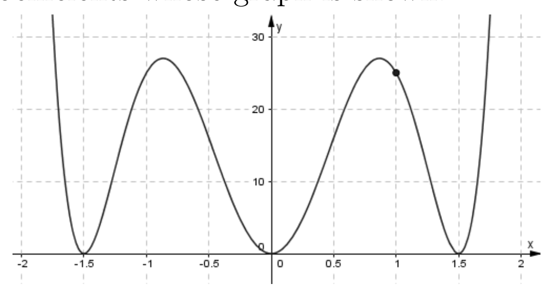
   (a) $f(x) = 2x^3 + x^2 - 3x + 1$.
   ANS: First we need to find a rational zero.

x				y
0	2	1	-3	1
1	2	3	0	1
$\frac{1}{2}$	2	2	-2	0

   So $f(x) = (2x-1)(x^2+x-1)$ and the zeros are at $\boxed{x = \frac{1}{2} \text{ and } \frac{-1 \pm \sqrt{5}}{2}}$

   (b) $p(x) = x^5 - 3x^3 + x = x(x^2+x-1)(x^2-x-1)$.
   ANS: Isn't it nice this puppy is factored down to quadratics?
   The zeros are now easy: $\boxed{x = 0, \frac{-1 \pm \sqrt{5}}{2}, \frac{1 \pm \sqrt{5}}{2}}$

2. Find a formula for the polynomial with integer coefficients whose graph is shown.



   (a) What does the $y-$axis symmetry tell you about the polynomial?
   ANS: The function is even, involving only even exponents in the expanded form.

   (b) What can you deduce about the polynomial from its behavior at (0,0)?
   ANS: There is a factor of $(x-0)^2 = x^2$

   (c) What does the root at $(1.5, 0)$ tell you about the polynomial (given there are integer coefficients.)?
   ANS: There is a factor of $(2x-3)^2$ (it's a double root).

   (d) Find an expression for the polynomial. *Hint*: it passes through $(1, 25)$.
   ANS: There's also a factor of $(2x+3)^2$, so all together, $f(x) = x^2((2x-3)(2x+3))^2 = x^2(4x^2-9)^2 = 16x^6 - 72x^4 + 81x^2$

3. Consider the rational function $R(x) = (4x^3 - 9x)/(x^3 - 1)$

   (a) What are the x-intercepts ?
   ANS: $x = 0, \pm\frac{3}{2}$

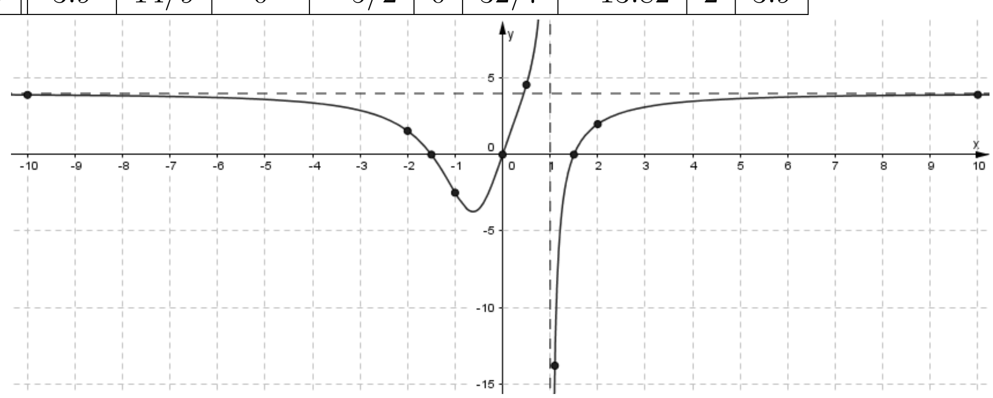
   (b) What is the y-intercept?
   ANS: $(0,0)$

   (c) What vertical asymptote(s) are there?
   ANS: $x = 1$

   (d) What is the horizontal asymptote?
   ANS: $y = 4$

   (e) Complete the table of values (approximate, as appropriate) and sketch a graph.

x	-10	-2	-1.5	-1	0	0.5	1.1	2	10
y	3.9	14/9	0	$-5/2$	0	32/7	-13.82	2	3.9



4. Solve each equation.

   (a) $\log_2(x^2-32)-\log_2(x+8)=1$

   ANS: $\log_2(x^2-32)-\log_2(x+8)=1 \Leftarrow \log_2\dfrac{x^2-32}{x+8}=1 \Leftrightarrow \dfrac{x^2-32}{x+8}=2 \Leftrightarrow$ $x^2-32=2x+16 \Leftrightarrow x^2-2x-48=0 \Leftrightarrow (x-8)(x+6)=0$ and both solutions work in the original equation, so $\boxed{x=8 \text{ or } x=-6}$

   (b) $4=\dfrac{10}{1+4e^{-0.8t}}$

   ANS: $4=\dfrac{10}{1+4e^{-0.8t}} \Leftrightarrow 4+16e^{-0.8t}=10 \Leftrightarrow e^{-0.8t}=\frac{3}{8} \Leftrightarrow -0.8t=\ln\frac{3}{8} \Leftrightarrow$ $\boxed{t=\frac{5}{4}\ln\frac{8}{3}\approx 1.2260365662646577960705639093 15}$

5. The half-life of polonium-210 is 138 days. Suppose we have a 100-g sample.

   (a) Find a function $m(t)=m_0 2^{-t/h}$ that models the mass remaining after t days.

   ANS: $\boxed{m(t)=100\cdot 2^{-t/138}}$

   (b) Find a function $m(t)=m_0 e^{-rt}$ that models the mass remaining after t days.

   ANS: $2^{-1/138}=e^r \Leftrightarrow r=-\dfrac{1}{138}\ln(2)\approx -0.00502280$ so $\boxed{m(t)=100e^{-0.00502t}}$

   (c) How much of the sample will remain after 400 days?

   ANS: $\boxed{m(400)=100e^{-2.009}\approx 13.41 \text{ g}}$

   (d) After how many days will only 20 g of the sample remain?

   ANS: $m(t)=20 \Leftrightarrow 100e^{-0.00502t}=20 \Leftrightarrow e^{-0.00502t}=0.2 \Leftrightarrow \boxed{t\approx \dfrac{\ln 0.2}{-0.00502}\approx 320.6 \text{ days}}$

6. For the angles $\alpha=\arctan(3/4), \beta=\arctan(\sqrt{3})$ simplify each of the following.

   (a) $\sin(\alpha+\beta)$.

   ANS: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha=\dfrac{3}{5}\cdot\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}\cdot\dfrac{4}{5}=\boxed{\dfrac{3+4\sqrt{3}}{10}}$

   (b) $\cos(2\alpha+\beta)$.

   ANS: $\cos(2\alpha+\beta)=\cos(2\alpha)\cos\beta-\sin(2\alpha)\sin\beta=(2\cos^2\alpha-1)\dfrac{1}{2}-2\sin\alpha\cos\alpha\dfrac{\sqrt{3}}{2}$ $\left(2\left(\dfrac{4}{5}\right)^2-1\right)\cdot\dfrac{1}{2}-2\cdot\dfrac{3}{5}\cdot\dfrac{4}{5}\cdot\dfrac{\sqrt{3}}{2}=\boxed{\dfrac{7}{50}-\dfrac{12\sqrt{3}}{25}\approx -0.6914}$

7. Find all solutions to each equation.

   (a) $8\sin^3(x)-4\sin^2(x)-6\sin(x)+3=0$ *Hint*: factor by grouping.

   ANS: $4\sin^2(x)(2\sin x-1)-3(2\sin(x)-1)=0 \Leftrightarrow (4\sin^2(x)-3)(2\sin(x)-1)=0$ So either $4\sin^2(x)-3=0 \Leftrightarrow \sin(x)=\pm\dfrac{\sqrt{3}}{2} \Leftrightarrow \boxed{x=\dfrac{\pi(6k+3\pm 1)}{6}}$ or $\sin(x)=\frac{1}{2} \Leftrightarrow$ $\boxed{x=\dfrac{\pi(6k+3\pm 1)}{3}}$

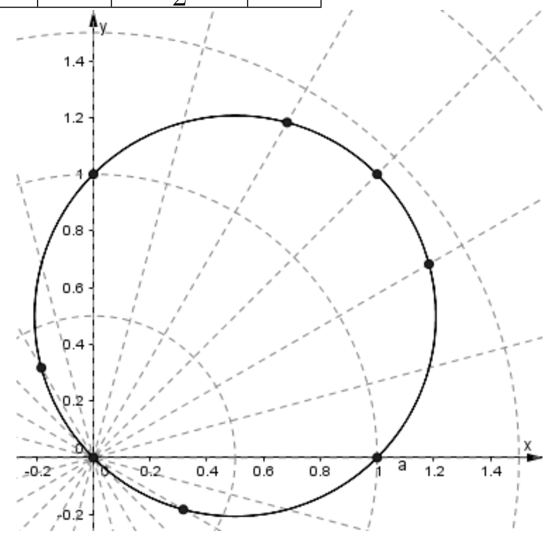
(b) $\sec\theta + \tan\theta = \frac{5}{3}$

$\sec\theta + \tan\theta = \frac{5}{3} \Leftrightarrow \sec\theta = \frac{5}{3} - \tan\theta \Rightarrow \tan^2\theta + 1 = \frac{25}{9} - \frac{10}{3}\tan\theta + \tan^2\theta \Leftrightarrow$
$\frac{10}{3}\tan\theta = \frac{16}{9} \Leftrightarrow \tan\theta = \frac{8}{15} \Leftrightarrow \boxed{\theta = \arctan\frac{8}{15} + k\pi}$

8. Complete the table of values and plot the polar function. $r = \sin\theta + \cos\theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	1	$\frac{1+\sqrt{3}}{2}$	$\sqrt{2}$	$\frac{1+\sqrt{3}}{2}$	1	$\frac{-1+\sqrt{3}}{2}$	0	$\frac{1-\sqrt{3}}{2}$	-1

You can convert this to rectangular form:
$r = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$, is a rotation by $-\frac{\pi}{4}$ of
$r = \sqrt{2}\sin\theta \Leftrightarrow r^2 = \sqrt{2}r\sin\theta \Leftrightarrow x^2 + y^2 = \sqrt{2}y \Leftrightarrow x^2 + \left(y - \frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$, a circle of radius $\frac{\sqrt{2}}{2}$, centered at $\left(0, \frac{\sqrt{2}}{2}\right)$



9. Consider the ellipse whose equation is $\frac{x^2}{9} + \frac{(y-2)^2}{16} = 1$

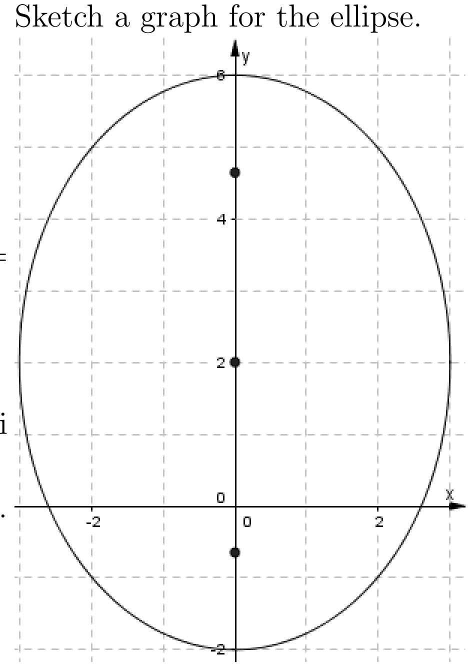
(a) Find the coordinates of center.
ANS: $(0, 2)$

(b) Find the x-intercepts of the ellipse.
ANS: If $y = 0$, then $\frac{x^2}{9} = \frac{3}{4} \Leftrightarrow x = \pm\frac{3\sqrt{3}}{2}$

(c) Find the coordinates of the two foci.
ANS: $c^2 = a^2 - b^2 = 16 - 9 = 7$ so the foci are at $(0, 2 \pm \sqrt{7})$

(d) Write parametric equations for the ellipse.
ANS: $x = 3\cos(t), y = 2 + 4\sin(t)$

(e) Sketch a graph for the ellipse.



10. Consider the parametric equations

$$x = 2\tan(t)$$
$$y = 3 + 4\sec(t)$$

(a) Eliminate the parameter to find an equation relating x and y directly. *Hint*: $\sec^2(t) - \tan^2(t) = 1$.

ANS: $\dfrac{(y-3)^2}{16} - \dfrac{x^2}{4} = \sec^2(t) - \tan^2(t) = 1$

(b) Tabulate values for t, x and y and use these to sketch a graph for the relation.

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	0	2	NAN	-2	0	2	NAN	-2	0
y	7	$3+4\sqrt{2}$	NAN	$3-4\sqrt{2}$	-1	$3-4\sqrt{2}$	NAN	$3+4\sqrt{2}$	7