Math 12 – Redemption Synthesis Test 1 – spring '09 Name________________________

Write all responses on separate paper. Show your work for credit.

1. Note that the exponential function $f(x) = e^x$ is often written as $f(x) = \exp(x)$.

   Let $p(x) = 1 - \exp\left(\left(3x^2 - 4\right)\left(x^2 - 3\right)\right)$. Find the x-intercepts of p.

2. Let $L(x) = \log_6\left(x^3 - 6x^2 + 11x - 6\right)$
   a. Solve for x: $L(x) = 1$.
   b. Where are the vertical asymptotes of $L(x)$?

3. Let $p(x) = 8x^4 - 6x^2 + 1$
   a. List all possible rational roots according tot the theorem on rational roots.
   b. Use synthetic division and the remainder theorem to show that $x = ½$ is a zero of p.
   c. Find all vertical asymptotes of the function $f(x) = \dfrac{1}{p(x)}$.

4. Find a polynomial with integer coefficients and zeros at $x = \dfrac{2}{5}$, $x = \dfrac{\sqrt{3}}{2}$ and $x = 2i$.

   Write the polynomial in descending powers form.

5. Let $p(x) = 2x^4 - 3x^2 + 1$.
   a. Find the 4 zeros of p. Write these in exact, simplified form.
   b. Find at least 4 different solutions to $p(\sin(t)) = 0$.

6. The future value of an investment P in an account yielding annual interest rate r after n years is given by the formula $V = \dfrac{P \cdot r}{1 - (1 + r)^n}$.

Math 12 – Redemption Synthesis Test 1 Solutions.

1. Note that the exponential function $f(x) = e^x$ is often written as $f(x) = \exp(x)$.

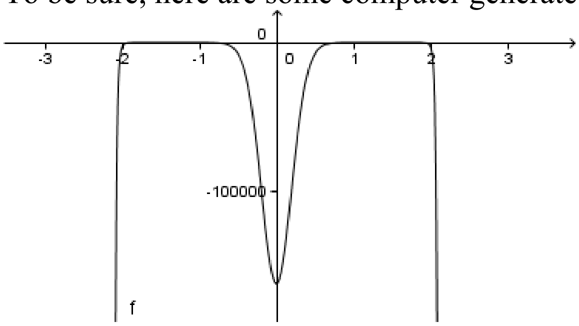
   Let $p(x) = 1 - \exp\left(\left(3x^2 - 4\right)\left(x^2 - 3\right)\right)$. Find the x-intercepts of p.

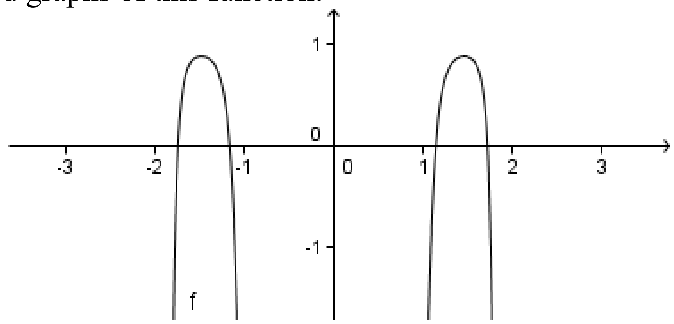
   SOLN: The x-intercepts are where

   $p(x) = 0 \Leftrightarrow \exp\left(\left(3x^2 - 4\right)\left(x^2 - 3\right)\right) = 1 \Leftrightarrow \left(3x^2 - 4\right)\left(x^2 - 3\right) = 0 \Leftrightarrow 3x^2 - 4 = 0 \text{ or } x^2 - 3 = 0$

   This means the x-intercepts are $\left(\frac{2\sqrt{3}}{3}, 0\right), \left(-\frac{2\sqrt{3}}{3}, 0\right), \left(\sqrt{3}, 0\right)$ and $\left(-\sqrt{3}, 0\right)$

   To be sure, here are some computer generated graphs of this function:





   These are actually the same graph with radically different scales on the vertical axis. This reminds me of the counter-intuitive notion that Earth is actually much smoother than a typical cue-ball in pool.

2. Let $L(x) = \log_6\left(x^3 - 6x^2 + 11x - 6\right)$

   a. Solve for x: $L(x) = 1$.

   SOLN: $L(x) = \log_6\left(x^3 - 6x^2 + 11x - 6\right) = 1 \Leftrightarrow x^3 - 6x^2 + 11x - 6 = 6^1 \Leftrightarrow x^3 - 6x^2 + 11x - 12 = 0$

   The rational zeros are in the set $\pm\{1, 2, 3, 4, 6, 12\}$ so using synthetic division (at right) we find that $x = 4$ is a zero so that $x^3 - 6x^2 + 11x - 12 = (x - 4)\left(x^2 - 2x + 3\right)$ and since the quadratic factor has discriminant = –8 < 0, the only x-intercept is (4,0)

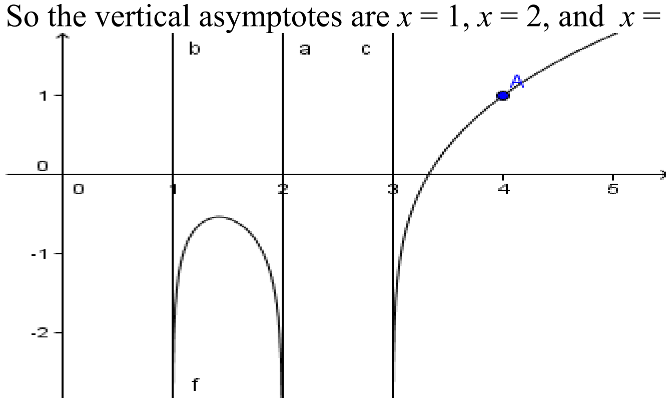
   The other zeros are where $x = 1 \pm \sqrt{2}\, i$

in				out
0	1	–6	11	–12
1	1	–5	6	–6
2	1	–4	3	–6
3	1	–3	2	–6
4	1	–2	3	0

   b. Where are the vertical asymptotes of $L(x)$?

   SOLN: The vertical asymptotes are where $x^3 - 6x^2 + 11x - 6 = 0$. The rational zeros are in the set $\pm\{1, 2, 3, 6\}$ $x^3 - 6x^2 + 11x - 6 = (x - 1)\left(x^2 - 5x + 6\right) = (x - 1)(x - 2)(x - 3) = 0 \Leftrightarrow x \in \{1, 2, 3\}$

   So the vertical asymptotes are $x = 1$, $x = 2$, and $x = 3$



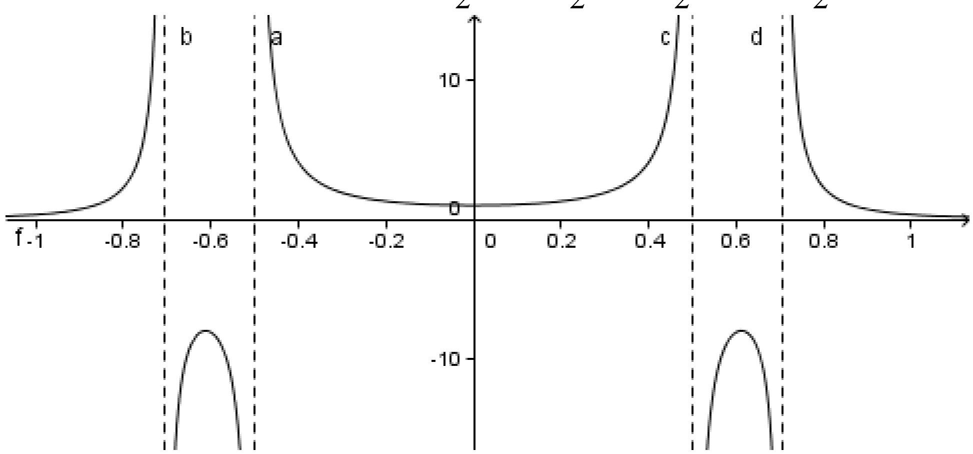
blunt

3. Let $p(x) = 8x^4 - 6x^2 + 1$
   a. List all possible rational roots according tot the theorem on rational roots.
      SOLN: $\pm\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\}$
   b. Use synthetic division and the remainder theorem to show that $x = ½$ is a zero of p.
      SOLN:

$1/2$	8	0	-6	0	1
		4	2	-2	-1
	8	4	-4	-2	0

      so $p(x) = (2x-1)(4x^3 + 2x^2 - 2x - 1)$
   c. Find all vertical asymptotes of the function $f(x) = \frac{1}{p(x)}$.
      SOLN: The other zeros must be zeros of the quotient, $4x^3 + 2x^2 - 2x - 1$. After some experimentation we find that -½ is also a zero so that $p(x) = (2x-1)(2x+1)(2x^2 - 1)$

      So the vertical asymptotes are $x = \frac{1}{2},\ x = -\frac{1}{2},\ x = \frac{\sqrt{2}}{2},\ x = -\frac{\sqrt{2}}{2}$

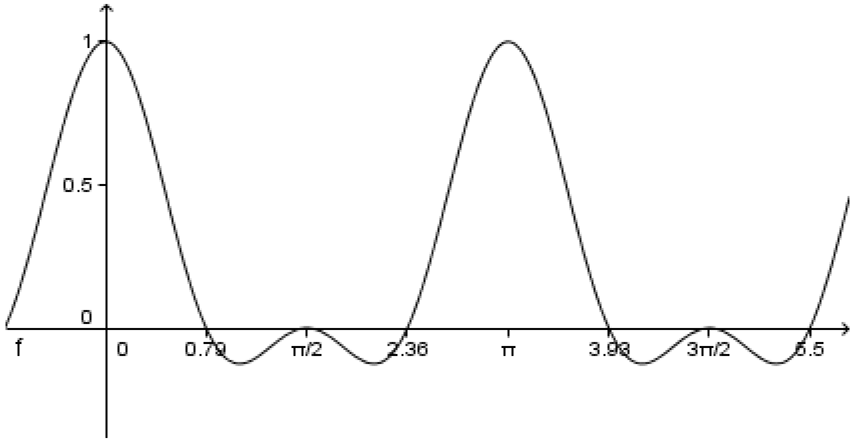


4. Find a polynomial with integer coefficients and zeros at $x = \frac{2}{5}$, $x = \frac{\sqrt{3}}{2}$ and $x = 2i$.
   Write the polynomial in descending powers form.
   SOLN: The factor $5x - 2$ will yield the rational zero. The irrational zero is accommodated by the factor $4x^2 - 3$ and the imaginary zero will result from a factor of $x^2 + 4$. All together, the polynomial is $(5x-2)(4x^2-3)(x^2+4) = (5x-2)(4x^4 + 13x^2 - 12) = 20x^5 - 8x^4 + 65x^3 - 26x^2 - 60x - 24$

5. Let $p(x) = 2x^4 - 3x^2 + 1$.
   a. Find the 4 zeros of p. Write these in exact, simplified form.
      SOLN: $p(x) = 2x^4 - 3x^2 + 1 = (2x^2 - 1)(x^2 - 1) = 2\left(x - \frac{\sqrt{2}}{2}\right)\left(x + \frac{\sqrt{2}}{2}\right)(x-1)(x+1)$

      So the solutions are $x = \pm\frac{\sqrt{2}}{2}$ and $x = \pm 1$
   b. Find at least 4 different solutions to $p(\sin(t)) = 0$.
      SOLN: We want either $\sin(t) = \pm\frac{\sqrt{2}}{2}$ or $\sin(t) = \pm 1$. Thus either $t = \frac{\pi}{4} + \frac{k\pi}{2} = \frac{(2k+1)\pi}{4}$ (any

odd multiple of π over 4. or $t = \frac{(2k+1)\pi}{2}$ (any odd multiple of π over 2.)



.