Math 12 – Precalculus – Exponential and Logarithm Functions Test – Fall '09 NAME________________

Write all responses on separate paper. Show all work for credit. You will need a scientific calculator.

1. Rewrite the equation $3.98+10^{1-x/2}=10$ in equivalent logarithmic form, then solve for x. Use a calculator to approximate x to the nearest ten thousandth.

2. Rewrite the equation $8-4\log(x-2)=6.09$ in equivalent exponential form, then solve for x. Use a calculator to approximate x to the nearest ten thousandth.

3. Solve the equation for x. Approximate to the nearest thousandth.
   a. $10^x = 6.67259\times 10^{-11}$
   b. $1.01^{x/24} = 7$

4. Use properties of logarithms to solve the equation. Approximate to 4 digits if appropriate.
   a. $\log_2(x)+\log_2(44-3x)=7$
   b. $\log\left(\frac{10}{9x^2}\right)+\log(1)=0.5$

5. Given $f(x)=1+1.322^x$.
   a. Find a formula for the inverse function, $y=f^{-1}(x)$.
   b. Find the intercept and asymptote of $f(x)$.
   c. Find the intercept and asymptote of $f^{-1}(x)$.
   d. Complete the table

x	-2	0	2	4	6	8	10
$f(x)$							

   and use the results to sketch a graphing showing $y=f(x)$ and $y=f^{-1}(x)$ together and illustrating the symmetry through $y=x$.

6. Suppose the Gorkon population on planet Xorda in April of 1999 was 1380, and it is estimated that the population will increase by 2% every 400 years.

   a. Assuming a natural growth model for the Gorkons, when will its population have grown to 2000?

   b. By what percentage will the population grow in 1800 years?

7. If \$300 is invested at 3.65% annual interest rate compounded daily, how long will it take to reach a value of \$500?

8. Actinium has a half life of about 7.04×10^8 years. How long will it take a 1 gram sample to decay to one milligram (one thousandth of a gram).

9. Find an exponential function (of the form $f(x)=a\bullet b^x$) which passes through the points (2,5) and (5,8). What is the value of $f(8)$?

Math 12 – Precalculus – Exponential and Logarithm Functions Test Solutions – fall ’09

1. Rewrite the equation $3.98+10^{1-x/2}=10$ in equivalent logarithmic form, then solve for *x*. Use a calculator to approximate *x* to the nearest ten thousandth.

   SOLN:
   $$3.98+10^{1-x/2}=10 \Leftrightarrow 10^{1-x/2}=6.02 \Leftrightarrow 1-x/2=\log(6.02) \Leftrightarrow x=2(1-\log(6.02))$$
   $$x \approx 2(1-0.77960)=2(0.22040)\approx 0.4408$$

2. Rewrite the equation $8-4\log(x-2)=6.09$ in equivalent exponential form, then solve for *x*. Use a calculator to approximate *x* to the nearest ten thousandth.

   SOLN:
   $$8-4\log(x-2)=6.09 \Leftrightarrow -4\log(x-2)=-1.91 \Leftrightarrow \log(x-2)=0.4775 \Leftrightarrow x-2=10^{0.4775}\approx 3.00262$$
   $$x \approx 5.0026$$

3. Solve the equation for *x*. Approximate to 4 digits, if appropriate.

   a. $10^x = 6.67259\times 10^{-11}$

      SOLN: $x=\log(6.67259)-11 \approx 0.824294-11=-10.175706$

   b. $1.01^{x/24}=7$

      SOLN: $1.01^{x/24}=7 \Leftrightarrow \ln\left(1.01^{x/24}\right)=\ln 7 \Leftrightarrow \frac{x}{24}\ln(1.01)=\ln 7 \Leftrightarrow x=\frac{24\ln 7}{\ln(1.01)}\approx 4693$

4. Use properties of logarithms to solve the equation. Approximate to 4 digits if appropriate.

   a. $\log_2(x)+\log_2(44-3x)=7$

      SOLN:
      $$\log_2(x)+\log_2(44-3x)=7 \Rightarrow \log_2(x(44-3x))=7 \Leftrightarrow -3x^2+44x=128$$
      $$\Leftrightarrow 3x^2-44x-128=0 \Leftrightarrow (3x-32)(x-4)=0 \Leftrightarrow \boxed{x=\frac{32}{3}} \text{ or } \boxed{x=4}$$

   b. $\log\left(\frac{10}{9x^2}\right)+\log(1)=0.5$ SOLN:
      $$\log\left(\frac{10}{9x^2}\right)+\log(1)=0.5 \Leftrightarrow \log 10-\log 9x^2+0=0.5 \Leftrightarrow \log 9x^2=\frac{1}{2} \Leftrightarrow 9x^2=\sqrt{10} \Leftrightarrow \boxed{x=\frac{\pm 10^{1/4}}{3}}$$

5. Given $f(x)=1+1.322^x$.

   a. Find a formula for the inverse function, $y=f^{-1}(x)$.

      SOLN: $y=1+1.322^x \Leftrightarrow 1.322^x=y-1 \Leftrightarrow \log\left(1.322^x\right)=\log(y-1) \Leftrightarrow x\log 1.322=\log(y-1)$
      $$x=\frac{\log(y-1)}{\log(1.322)}\approx 8.2487\log(y-1) \Leftrightarrow \boxed{f^{-1}(x)=8.2487\log(x-1)}$$

   b. Find the intercept and asymptote of $f(x)$.

      SOLN: The graph of $y=f(x)$ rises from a horizontal asymptote *y* = 1 through the *y*-intercept at (0,2).

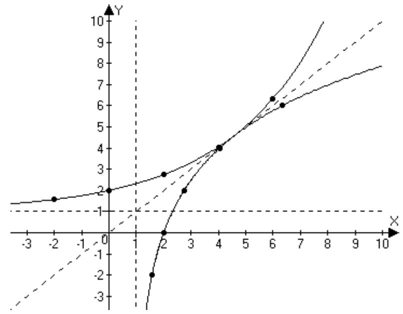
   c. Find the intercept and asymptote of $f^{-1}(x)$.

      SOLN: The graph of $y=f^{-1}(x)$ rises along a vertical asymptote *x* = 1 through the *x*-intercept at (2,0).

d. ANS: Completing the table we have

x	-2	0	2	4	6	8	10
$f(x)$	1.57	2	2.75	4.05	6.34	10.3	17.3

and thus a graphing showing $y = f(x)$ and $y = f^{-1}(x)$ together illustrates the symmetry through $y = x$:



6. Suppose the Gorkon population on planet Xorda in April of 1999 was 1380, and it is estimated that the population will increase by 2% every 400 years.
   a. Assuming a natural growth model for the Gorkons, when will its population have grown to 2000?
   SOLN: According to the natural growth model, the population t years from April of 1999 will be $P(t) = 1380(1.02)^{t/400} \approx 1380e^{0.00004951t}$. Setting either of these to 2000 and solving for t we have $(1.02)^{t/400} = \frac{200}{138} \Leftrightarrow \frac{t}{400}\ln(1.02) = \ln\left(\frac{100}{69}\right) \Leftrightarrow t = 400\left(\frac{\ln 100 - \ln 69}{\ln 1.02}\right) \approx 7495.24$ Add this to the base year and you've got about 1999+7495 = 9494 and a quarter year from April is July.
   b. By what percentage will the population grow in 1800 years?
   SOLN: $P(1800) = 1380(1.02)^{9/2} \approx 1380(1.0932)$, so the population will grow 9.32%
7. If \$300 is invested at 3.65% annual interest rate compounded daily, how long will it take to reach a value of \$500?
   SOLN:
   $$300\left(1+\frac{0.0365}{365}\right)^{365t} = 500 \Leftrightarrow (1.0001)^{365t} = 5/3 \Leftrightarrow 365t = \frac{\ln 5 - \ln 3}{\ln 1.0001} \Leftrightarrow t = \frac{\ln 5 - \ln 3}{365\ln 1.0001} \approx 13.996 \text{ yrs}$$
   Note that the continuous compounding model yields an extremely close result:
   $300e^{0.0365t} = 500 \Leftrightarrow e^{0.0365t} = 5/3 \Leftrightarrow 0.0365t = \ln(5/3) \Leftrightarrow t = \ln(5/3)/0.0365 \approx 13.995$
8. Actinium has a half life of about 7.04×10^8 years. How long will it take a 1 gram sample to decay to one milligram (one thousandth of a gram).
   SOLN: If 1 gram of Actinium is present at t = 0, then the amount which remains at time t (in years) then solve
   $A(t) = (0.5)^{t/7.04\times 10^8} = 0.001 \Leftrightarrow t = 7.04\times 10^8 \ln(0.001)/\ln(0.5) \approx 7.016\times 10^9$
9. Find an exponential function (of the form $f(x) = a \bullet b^x$) which passes through the points (2,5) and (5,8). What is the value of f(8) ?
   SOLN: $a\bullet b^2 = 5$ and $a\bullet b^5 = 8$ so $\frac{a\bullet b^5}{a\bullet b^2} = \frac{8}{5} \Leftrightarrow b^3 = \frac{8}{5} = 1.6 \Leftrightarrow b = \sqrt[3]{1.6}$ whence
   $a\bullet\sqrt[3]{1.6}^2 = 5 \Leftrightarrow a = \frac{5}{\sqrt[3]{2.56}}$ and $f(8) = \frac{5}{\sqrt[3]{1.6}^2}\bullet\sqrt[3]{1.6}^8 = 5\sqrt[3]{1.6}^6 = 5(2.56) = 12.8$
   The simpler way to get this here is to note that the x increments from 2 to 5 and then to 8 are the same, so we can apply the same growth multiplier: 8 = 5*(8/5) and so 12.8 = 8*(8/5).
10. A potato is heated to 165°C and thrown into a huge bucket of ice water (0°C) so that two minutes later, its temperature is 100°C. How long will it take to cool to 10°C?
    SOLN: Since the ice water temperature is 0, Newton's law of cooling reduces to simple exponential decay: $T(t) = 165e^{-kt}$. Find k by plugging in the known values:
    $T(2) = 165e^{-2k} = 100 \Leftrightarrow k = -\ln(20/33)/2 \approx 0.25039$ To find when the potato temperature is 10°C, solve $165\exp(\ln(20/33)t/2) = 10 \Leftrightarrow \ln(20/33)t/2 = \ln(2/33) \Leftrightarrow t = 2\ln(2/33)/\ln(20/33) \approx 11.2$ minutes.