Math 12 – Chapter 8 Test – spring '09 Name________________________

Write all responses on separate paper. Show your work for credit.

1. Graph the polynomial $p(x) = -(x^2-4)(x^2-9)$ showing clearly all x and y intercepts and the end behavior.

2. Use synthetic division to show that 7 and -2 are both roots of $x^4 - 6x^3 - 11x^2 + 24x + 28$

3. Let $p(x) = 12x^4 - 3x^3 - 19x^2 + x + 5$
   a. List all possible rational roots according tot the theorem on rational roots.
   b. Explain why there must be a zero between $x = 0$ and $x = 1$.
   c. Show that $x = -1$ and $x = 5/4$ are zeros
   d. Find the other 2 zeros.

4. Let $p(x) = (x-1)^2(2x+5)(2x-5)$. Sketch a graph of $p(x)$ showing its behavior near its zeros and its end behavior.

5. Find a polynomial with integer coefficients and zeros at $x = 1.5$, $x = 1+\sqrt{2}$ and $x = 1+3i$.
   Write the polynomial in descending powers form.

6. Let $p(x) = 36x^4 - 60x^3 - 47x^2 + 60x + 16$.
   a. What does Descartes' Rule of Signs say about the number of positive zeros and the number of negative zeros?
   b. Show that $x = 3$ is an upper bound on the real zeros of p.

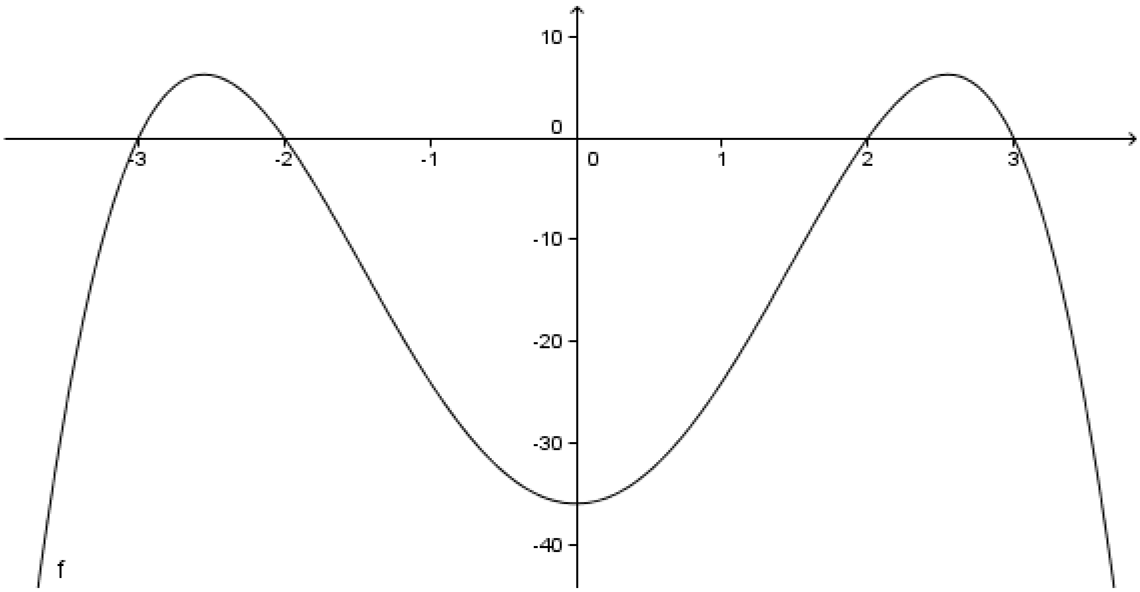
7. Consider the rational function $f(x) = \dfrac{(2x-1)(x+2)}{(3x-2)(2x+3)}$
   a. Find all intercepts.
   b. Find all vertical asymptotes.
   c. Find an equation for the horizontal asymptote.
   d. Make a table of at least 6 points on the graph and construct a graph showing the asymptotes and intercepts.

8. Find a rational function with a horizontal asymptote along $y = 2$,
   x-intercepts at (–3,0) and (2,0) and no vertical asymptotes.

Math 12 – Chapter 8 Test Solutions – spring ’09

1. Graph the polynomial $p(x) = -(x^2 - 4)(x^2 - 9)$ showing clearly all *x* and *y* intercepts and the end behavior.

   SOLN:



2. Use synthetic division to show that 7 and -2 are both roots of $x^4 - 6x^3 - 11x^2 + 24x + 28$

   SOLN:

IN					OUT
0	1	-6	-11	24	28
7	1	1	-4	-4	0
-2	1	-8	5	14	0

   Further, it’s important to be able to interpret these results:

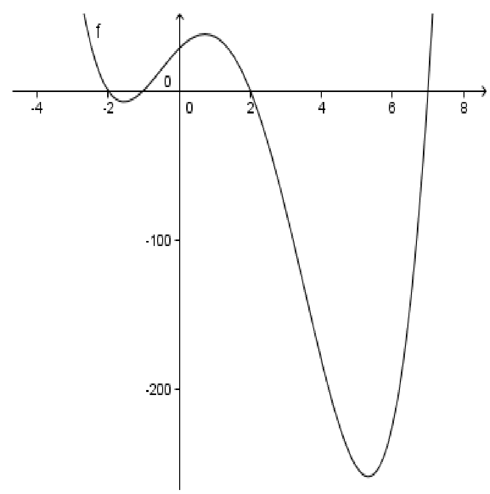
   $$x^4 - 6x^3 - 11x^2 + 24x + 28 = (x-7)(x^3 + x^2 - 4x - 4)$$

   $$= (x+2)(x^3 - 8x^2 + 5x - 14)$$

   and if you were to divide the quotient by *x* + 2 the second time, you’d get

   $$x^4 - 6x^3 - 11x^2 + 24x + 28 = (x-7)(x+2)(x^2 - x - 2) = (x-7)(x^2-4)(x+1)$$

   whose graph is a less symmetric and upwards opening W-shape:



3. Let $p(x) = 12x^4 - 3x^3 - 19x^2 + x + 5$
   a. List all possible rational roots according tot the theorem on rational roots.

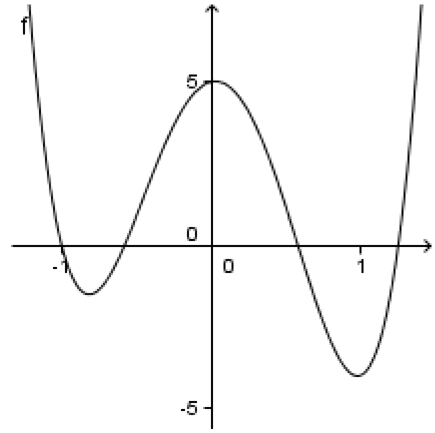
      SOLN: The rational zeros must be contained in $\left\{ \pm x \mid x \in \left\{ 1, 5, \frac{1}{2}, \frac{5}{2}, \frac{1}{3}, \frac{5}{3}, \frac{1}{4}, \frac{5}{4}, \frac{1}{6}, \frac{5}{6}, \frac{1}{12}, \frac{5}{12} \right\} \right\}$
   b. Explain why there must be a zero between *x* = 0 and *x* = 1.

      SOLN: $p(0) = 5$ and $p(1) = 12 - 3 - 19 + 1 + 5 = -4$ have opposite signs so, by the intermediate value theorem, there must be zero between *x* = 0 and *x* = 1.

c. Show that $x = -1$ and $x = 5/4$ are zeros

SOLN:

IN					OUT
0	12	-3	-19	1	5
-1	12	-15	-4	5	0
5/4	12	0	-4	0	



d. Find the other 2 zeros.

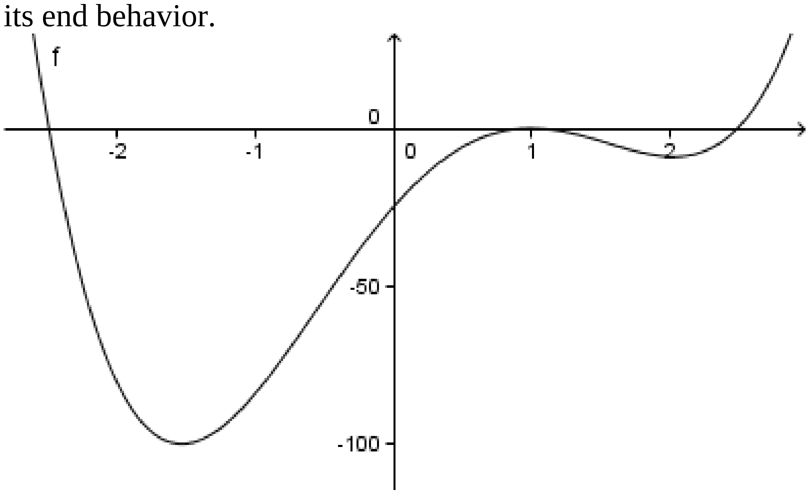
SOLN: As we can see from the above division:

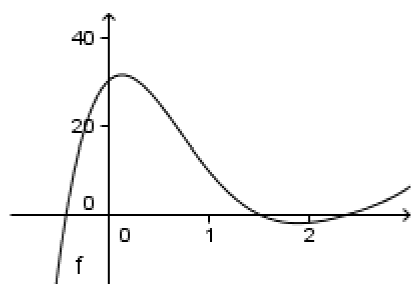
$$p(x) = (12x^2 - 4)\left(x - \frac{5}{4}\right)(x+1)$$

$$= (3x^2 - 1)(4x - 5)(x+1)$$

thus the other two zeros are zeros of $3x^2 - 1 = 0 \Leftrightarrow x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

4. Let $p(x) = (x-1)^2(2x+5)(2x-5)$. Sketch a graph of $p(x)$ showing its behavior near its zeros and its end behavior.





5. Find a polynomial with integer coefficients and zeros at $x = 1.5$, $x = 1 + \sqrt{2}$ and $x = 1 + 3i$. Write the polynomial in descending powers form.

SOLN: By the theorem of conjugates, the polynomial also has zeros at $1 - \sqrt{2}$ and $1 - 3i$

Using the factor theorem we can then write, regroup and use diff. of squares to expand:

$$(2x-3)\left[x - (1+\sqrt{2})\right]\left[x - (1-\sqrt{2})\right]\left[x - (1+3i)\right]\left[x - (1-3i)\right]$$

$$= (2x-3)\left[(x-1) - \sqrt{2}\right]\left[(x-1) + \sqrt{2}\right]\left[(x-1) - 3i\right]\left[(x-1) + 3i\right]$$

$$= (2x-3)\left[(x-1)^2 - 2\right]\left[(x-1)^2 + 9\right] = (2x-3)(x^2 - 2x - 1)(x^2 - 2x + 10)$$

$$= (2x-3)(x^4 - 4x^3 + 13x^2 - 18x - 10) = 2x^5 - 11x^4 + 38x^3 - 75x^2 + 34x + 30$$

Note the small sketch to the upper right shows the three real zeros, but not the complex zeros.

6. Let $p(x) = 36x^4 - 60x^3 - 47x^2 + 60x + 16$.

   a. What does Descartes' Rule of Signs say about the number of positive zeros and the number of negative zeros?

   SOLN: There are two sign changes in the coefficients as read in order descending powers, therefore there are either 2 positive zeros or none.
   $p(-x) = 36x^4 + 60x^3 - 47x^2 - 60x + 16$ also has two sign changes in the coefficients so there are also either 2 negative zeros or none.

   b. Show that $x = 3$ is an upper bound on the real zeros of p.
   SOLN: $p(x) = (x-3)(36x^3 + 48x^2 + 97x + 351) + 1069$ so that if $x > 3$ then $p(x)$ is a sum of two positive numbers, which can't be zero.

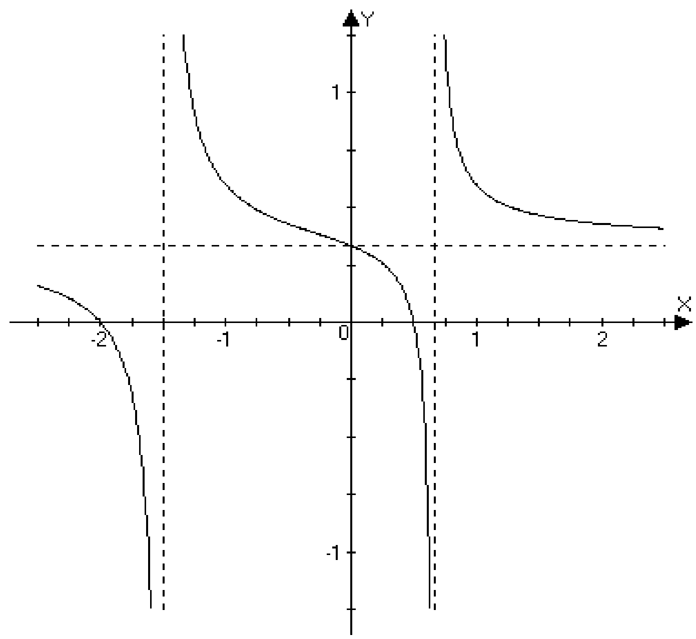
7. Consider the rational function $f(x) = \dfrac{(2x-1)(x+2)}{(3x-2)(2x+3)}$

   a. Find all intercepts.
   SOLN: The intercepts are
   $(-2,0), \left(0, \frac{1}{3}\right), \left(\frac{1}{2}, 0\right)$

   b. Find all vertical asymptotes.
   SOLN: The vertical asymptotes are along $x = 2/3$ and $x = -3/2$.

   c. Find an equation for the horizontal asymptote.
   SOLN: As $|x| \to \infty$, y approaches the line $y = 1/3$.

   d. Make a table of at least 6 points on the graph and construct a graph showing the asymptotes and intercepts.



x	$-5/2$	$-7/4$	-1	0	$6/10$	$7/10$
y	$3/19$	$-9/29$	$3/5$	$1/3$	$-13/21$	$27/11$

8. Find a rational function with a horizontal asymptote along $y = 2$, x-intercepts at $(-3,0)$ and $(2,0)$ and no vertical asymptotes.

SOLN: $\dfrac{2(x+3)(x-2)}{x^2+1}$ will do it.