Math 12 – Chapter 3 Test – Spring '05 Name_____
Show your work for credit. Write all responses on separate paper.

1. Consider the polynomial function $f(x) = 3x^4 + 2x^3 + 3x^2 - 4x - 4$.

   a. Use the leading term to describe the long-term behavior of $f(x)$; what happens as $|x| \to \infty$?

   b. Evaluate $f(1)$ and use the result to find a factor of $f(x)$.

   c. Use synthetic division to divide $f(x)$ by $x - 1$ and relate dividend, divisor, quotient and remainder in an equation.

   d. Compute the values of $f(-1)$ and $f(0)$. What theorem can you use to draw what conclusion about the location of a zero for $f(x)$?

   e. State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list possible rational zeros for $f(x)$.

   f. Use synthetic division to divide $f(x)$ by $x+2/3$ and relate dividend, divisor, quotient and remainder in an equation.

   g. Use long division to divide $f(x)$ by $x^2 + x + 2$ and relate dividend, divisor, quotient and remainder in an equation.

   h. Find the complex conjugate zeros of $x^2 + x + 2$.

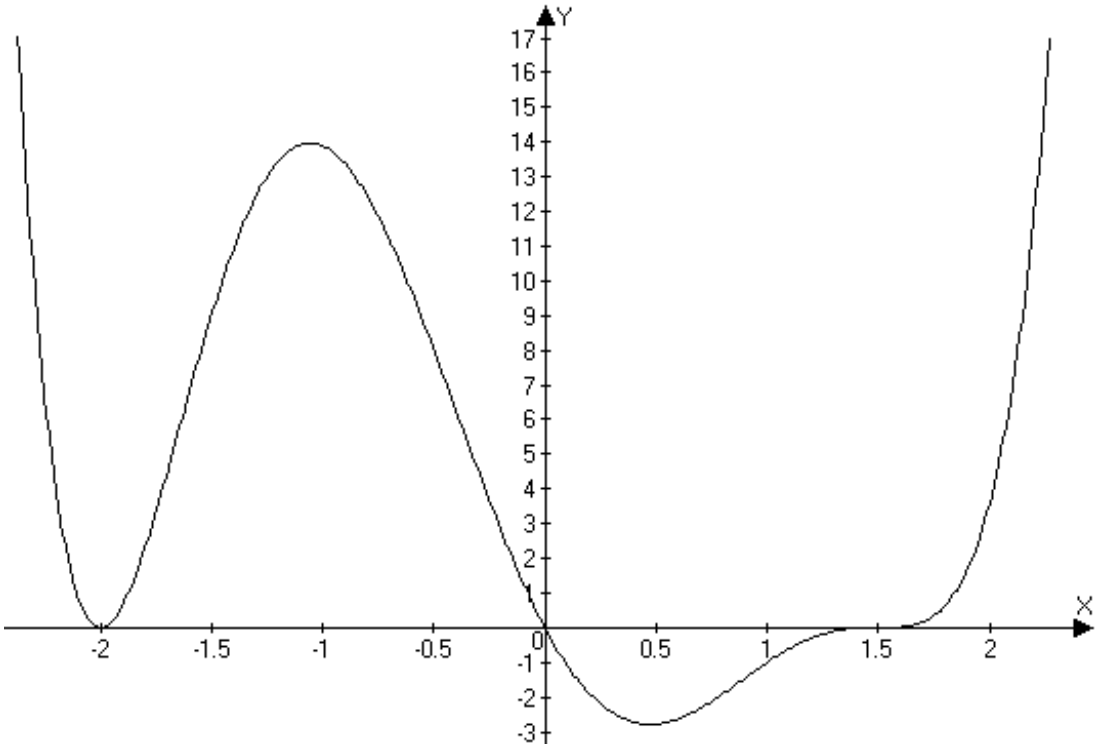
   i. Write $f(x) = 3x^4 + 2x^3 + 3x^2 - 4x - 4$ in completely factored form.

   j. Sketch a graph of $f(x) = 3x^4 + 2x^3 + 3x^2 - 4x - 4$ showing how it passes through its intercepts and the coordinates of all local or global extrema.

2. Explain why, if $x > 5$ then $p(x) = (x-5)(x^5 + x^3 + 1) + 1 > 1$ and thus $x = 5$ is an upper bound on the zeros of p.

3. Find a polynomial function with integer coefficients and zeros at $x = \frac{2}{3}$ and $x = -3 + \sqrt{2}i$.

4. Find a formula for the 6th degree polynomial whose graph is shown. Hint: it passes through (1,-1) and the leading coefficient is less than 1.



5. Sketch graphs for each of the following showing all intercepts, asymptotes and additional points, as necessary to get an approximate shape.

a. $y = \dfrac{x^2 - 9}{x\left(x^2 - 1\right)}$

b. $y = \dfrac{\left(x^2 - 4\right)\left(2x^2 - 1\right)}{\left(x - 1\right)^2\left(x^2 + x + 1\right)}$