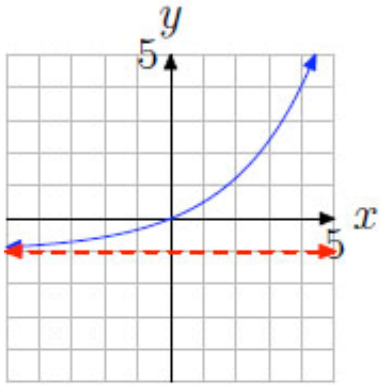
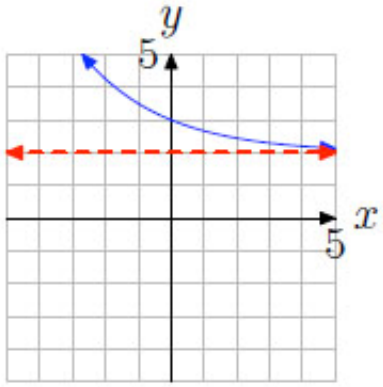
Math 12 – Chapter 10 Test – Spring '09 Name_____
Write all responses on separate paper. Show your work for credit.

1. For each, find an equation for the exponential function of the form $f(x) = c + a \cdot b^x$ whose graph is shown. Approximate to the nearest tenth, as needed.

   a.



   b.



   Hint: in the first the horizontal asymptote is $y = -1$ and in the second it appears that $f(-1) = 3.5$

2. Suppose that you invest $15,000 at 7% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

3. Suppose that you invest $16,000 at 4% interest compounded continuously. How much money will be in your account in 6 years? Round your answer to the nearest cent.

4. For each, find the exact value $f(b)$ of the given function f at the given value b.

   a. $f(x) = \log_3(x)$; $b = \sqrt[5]{3}$

   b. $f(x) = \log_5(x)$; $b = \dfrac{1}{25}$

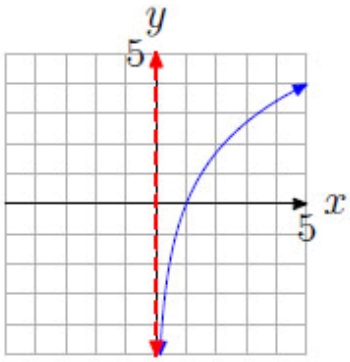
5. For each, use a calculator to evaluate the function at given value p. Round your answer to the nearest hundredth.

   a. $F(x) = \ln(x)$; $p = 10.06$

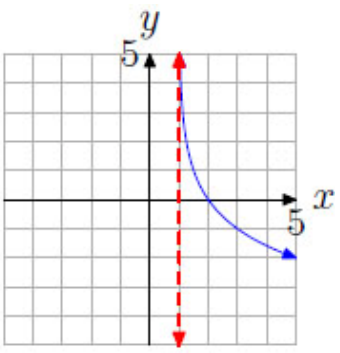
   b. $F(x) = \log(x)$; $p = 86.19$

6. For each, find an equation for the exponential function of the form $f(x) = \log_b(x - c)$ whose graph is shown. Approximate to the nearest tenth, as necessary.

   a.



   b.



   Hint: in the first the vertical asymptote is $x = 0$ and in the second it appears that $f(3) = -1$

7. Suppose that you invest $15,000 at 9% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

8. Use a calculator to evaluate the given function at the given value, round your answer to the nearest hundredth.
   a. $F(x) = \log_4(x)$; $p = 57.60$
   b. $F(x) = \log_8(x)$; $p = 302.67$

9. Suppose that you invest $16,000 at 2% interest compounded continuously. How many years will it take for your investment to reach $25,000? Round your answer to the nearest hundredth.

10. Solve for x: $\log_2(x-2) + \log_2(3x-8) = 3$.

11. Solve for t: $10\left(1+\frac{0.03}{24}\right)^{24t} = 20$. Approximate the solution to 4 digits.

12. Suppose that the population of a certain town grows at an annual rate of 3%. If the population grows to 2,000 in 7 years, what was the original population? Round your answer to the nearest integer.

13. Suppose that a certain radioactive isotope has an annual decay rate of 18.6%. If a particular sample decays to 41 grams after 3 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.

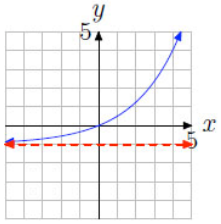
14. Find an exponential function of the form $P(t) = P_0 e^{rt}$ which passes through the points (0,7) and (2,14).

15. Let $f(x) = 4\log_2(x-3)$. Find a formula for the inverse function and graph the function together with the inverse showing the symmetry through the line $y = x$.

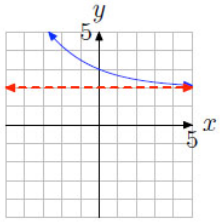
Math 12 – Chapter 10 Test Solutions – Spring '09

1. For each, find an equation for the exponential function of the form $f(x) = c + a \cdot b^x$ whose graph is shown. Approximate to the nearest tenth, as needed.

a.



b.



SOLN: (a) Note that $c = -1$ is the horizontal asymptote and that (1, 0.5) is an approximate value on the curve. Assuming $a = 1$, for simplicity, then we require $f(1) = -1 + b = 0.5 \Rightarrow b = 1.5$ so that $f(x) = -1 + 1.5^x$. As a check, note that $f(4) = -1 + 1.5^4 = 4.0625$

SOLN (b) $f(-1) = 3.5$ and the horizontal asymptote is $y = 2$, so

$f(-1) = 2 + b^{-1} = 3.5 \Leftrightarrow \frac{1}{b} = 1.5 \Leftrightarrow b = \frac{2}{3}$, thus $f(x) = 2 + \left(\frac{2}{3}\right)^x$ does the trick. Note that $f(2) = 2.4444\ldots$

2. Suppose that you invest \$15,000 at 7% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

SOLN: $15000\left(1 + \frac{0.07}{12}\right)^{12(4)} = 15000\left(1.00583\right)^{48} \approx 15000(1.32208387788) \approx \boxed{\$19830.81}$

3. Suppose that you invest \$16,000 at 4% interest compounded continuously. How much money will be in your account in 6 years? Round your answer to the nearest cent.

SOLN: $16000e^{0.04(6)} = 16000e^{0.24} \approx 16000(1.27124915032) \approx \boxed{\$20339.99}$

4. For each, find the exact value $f(b)$ of the given function f at the given value b.

a. $f(x) = \log_3(x);\ b = \sqrt[5]{3}$ SOLN: $f\left(\sqrt[5]{3}\right) = \log_3\left(\sqrt[5]{3}\right) = \frac{1}{5}$

b. $f(x) = \log_5(x);\ b = \frac{1}{25}$ SOLN: $f\left(\frac{1}{25}\right) = \log_5\left(5^{-2}\right) = -2$

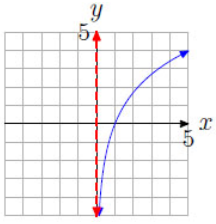
5. For each, use a calculator to evaluate the function at given value p. Round your answer to the nearest hundredth.

a. $F(x) = \ln(x);\ p = 10.06$ SOLN: $F(10.06) = \ln(10.06) \approx 2.31$

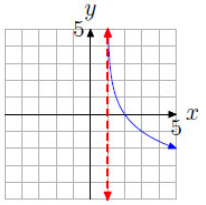
b. $F(x) = \log(x);\ p = 86.19$ SOLN: $F(86.19) = \log(86.19) \approx 1.94$

6. For each, find an equation for the exponential function of the form $f(x) = \log_b(x - c)$ whose graph is shown. Approximate to the nearest tenth, as necessary.

a.



b.



SOLN: (a) Since $x = 0$ is the vertical asymptote and $\log_b(1.5) = 1 \Leftrightarrow b = 1.5 \Rightarrow \boxed{f(x) = \log_{1.5}(x)}$

(b) Here $x = 1$ is a vertical asymptote and it appears that $f(3) = \log_b(3-1) = -1 \Leftrightarrow b^{-1} = 2 \Leftrightarrow b^{-1} = \frac{1}{2}$ so

$\boxed{f(x) = \log_{0.5}(x-1)}$

7. Suppose that you invest \$15,000 at 9% interest compounded monthly. How much money will be in your account in 4 years? Round your answer to the nearest cent.

SOLN: SOLN: $15000\left(1 + \frac{0.09}{12}\right)^{12(4)} = 15000\left(1.0075\right)^{48} \approx 15000(1.43140533331) \approx \boxed{\$21471.08}$

8. Use a calculator to evaluate the given function at the given value, round your answer to the nearest hundredth.

   a. $F(x) = \log_4(x)$; $p = 57.60$ SOLN: $F(57.60) = \log_4(57.60) = \dfrac{\ln(57.60)}{\ln 4} \approx \dfrac{4.0535225677}{1.38629436112} \approx 2.92$

   b. $F(x) = \log_8(x)$; $p = 302.67$ SOLN: $F(302.67) = \log_8(302.67)$ is approximately 2.75

9. Suppose that you invest \$16,000 at 2% interest compounded continuously. How many years will it take for your investment to reach \$25,000? Round your answer to the nearest hundredth.
   SOLN: $16000e^{0.02t} = 25000$ is equivalent to
   $\Leftrightarrow e^{0.02t} = \dfrac{25}{16} \Leftrightarrow 0.02t = \ln\left(\dfrac{25}{16}\right) \Leftrightarrow t = 100\ln\left(\dfrac{5}{4}\right) \approx 22.314$ years

10. Solve for x: $\log_2(x-2) + \log_2(3x-8) = 3$.
    SOLN: $\log_2(x-2)(3x-8) = 3 \Leftrightarrow (x-2)(3x-8) = 2^3 \Leftrightarrow 3x^2 - 14x + 8 = 0 \Leftrightarrow (x-4)(3x-2) = 0$ so $x = 4$ is the only solution, since the logarithms are not real-valued at $x = 2/3$.

11. Solve for t: $10\left(1 + \dfrac{0.03}{24}\right)^{24t} = 20$. Approximate the solution to 4 digits.
    SOLN: $(1.00125)^{24t} = 2 \Leftrightarrow t = \dfrac{\ln 2}{24\ln(1.00125)} \approx 23.12$

12. Suppose that the population of a certain town grows at an annual rate of 3%. If the population grows to 2,000 in 7 years, what was the original population? Round your answer to the nearest integer.
    SOLN: $P(t) = P_0e^{0.03t}$ is the model and $P(7) = P_0e^{0.21} = 2000 \Leftrightarrow P_0 = \dfrac{2000}{e^{0.21}} \approx 1621$

13. Suppose that a certain radioactive isotope has an annual decay rate of 18.6%. If a particular sample decays to 41 grams after 3 years, how big (in grams) was the original sample? Round your answer to the nearest hundredth.
    SOLN: $A(3) = A_0e^{-0.186(3)} = 41 \Leftrightarrow A_0 = \dfrac{41}{e^{-0.558}} = 71.63$ grams to start with.

14. Find an exponential function of the form $P(t) = P_0e^{rt}$ which passes through the points (0,7) and (2,14).
    SOLN: $P(0) = 7 \Rightarrow P(2) = 7e^{2r} = 14 \Leftrightarrow e^{2r} = 2 \Leftrightarrow r = \dfrac{1}{2}\ln 2 = \ln\sqrt{2}$ so
    $P(t) = 7e^{t\ln\sqrt{2}} = 7\left(e^{\ln\sqrt{2}}\right)^t = 7\left(\sqrt{2}\right)^t$

15. Let $f(x) = 4\log_2(x-3)$. Find a formula for the inverse function and graph the function together with the inverse showing the symmetry through the line $y = x$.
    SOLN:
    $y = 4\log_2(x-3) \Leftrightarrow x - 3 = 2^{y/4} \Leftrightarrow$
    $x = 3 + \left(\sqrt[4]{2}\right)^y \Leftrightarrow f^{-1}(x) = 3 + \left(\sqrt[4]{2}\right)^x$