Math 12 — Precalculus – Chapter 7 Test – Spring '09 Name________________________________

Instructions: Show your work for credit. Write all responses on separate paper. No calculators.

1. Solve these decimal fraction conversion problems:
   a. Write the fractions $\frac{1}{7},\frac{2}{7},\frac{3}{7}$ and $\frac{4}{7}$ as repeating decimals, being sure to indicate the repeating part by putting a bar over it.
   b. Show how to find the rational representation (ie, the ratio integers in lowest terms) of the repeating decimal, $0.243243243\cdots = 0.\overline{243}$

2. Simplify each, that is, write an equivalent expression without negative exponent and without complex fractions.
   a. $\dfrac{2x^3-2}{\left(x^2+x+1\right)\left(x-1\right)}$
   b. $\dfrac{\dfrac{8}{5\left(x+h\right)-2}-\dfrac{8}{5x-2}}{h}$

3. Find all solutions to each equation:
   a. $\left(x-1\right)^2=\dfrac{4}{3}$
   b. $\dfrac{1}{x-1}-\dfrac{1}{x^2-1}=\dfrac{5}{4\left(x+1\right)}$

4. Jamal has a collection of nickels, dimes and quarters, which, all together, weighs 93.9 grams. Dimes weigh 2.3 grams; nickels weigh 5.0 grams and the quarters, 5.7 grams. If the number of dimes is 3 more than the number of quarters, and the number of nickels is the number of quarters and dimes combined, how many quarters are there? Use the algebraic method to solve this problem.

5. The height of an isosceles triangle is three times the length of its base. If the area of the triangle is 12 square meters, what is the height of the triangle? Write your answer in simplest radical form and then approximate to the nearest hundredth of a meter.

6. Solve the inequality $\dfrac{1}{x-2}+\dfrac{3x}{2x-1}\le\dfrac{9}{10}$. Write the solution using interval notation.

7. Find an equation for the circle centered at (3,4) and passing through (0,0).

8. Find an equation for the line passing through (0,0) and perpendicular to the line $2x+3y=5$.

9. Pharmaceutical

Math 12 — Precalculus – Chapter 7 Test Solutions – Spring ‘09

1. Solve these decimal fraction conversion problems:

   a. Write the fractions $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$ and $\frac{4}{7}$ as repeating decimals, being sure to indicate the repeating part by putting a bar over it.

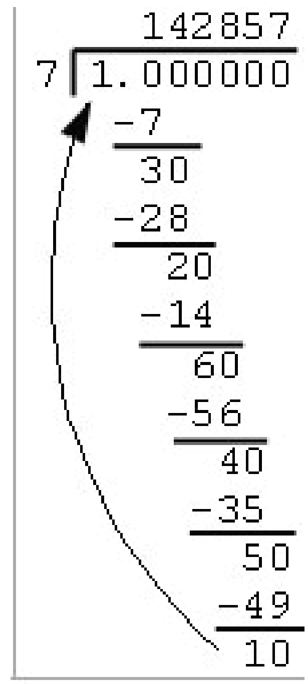
   SOLN: See the long division at right for the expression $\frac{1}{7} = 0.\overline{142857}$

   Notice that the division uses all possible non-zero remainders less than 7: 3, 4, 6, 4, 5, 1, respectively, so that that, with the remainder 1 the process is back to the beginning and will loop through the same sequence of quotients ad infinitum.

   Now all you need do is multiply by 2, 3 and 4 to get $\frac{2}{7} = 0.\overline{285714}$,

   $\frac{3}{7} = 0.\overline{428571}$ and $\frac{4}{7} = 0.\overline{571428}$

   It’s interesting to note that the decimals cycle through the digits in the same order, but with a different starting place. This is not unique to 7, but 7 is small enough to make it obvious and large enough to make it interesting.



   b. Show how to find the rational representation (ie, the ratio integers in lowest terms) of the repeating decimal, $0.243243243\cdots = 0.\overline{243}$

   $$1000\left(0.\overline{243}\right) - 0.\overline{243} = 243 \Leftrightarrow 999\left(0.\overline{243}\right) = 243 \Leftrightarrow 0.\overline{243} = \frac{243}{999} = \frac{9}{37}$$

2. Simplify each, that is, write an equivalent expression without negative exponent and without complex fractions.

   a. $\dfrac{2x^3 - 2}{\left(x^2 + x + 1\right)\left(x - 1\right)} = 2$,
   provided x is not 1.

   b. $$\frac{\dfrac{8}{5(x+h)-2} - \dfrac{8}{5x-2}}{h} \frac{\left(5(x+h)-2\right)(5x-2)}{\left(5(x+h)-2\right)(5x-2)}$$
   $$= \frac{8(5x - 2 - 5x - 5h + 2)}{h\left(5(x+h)-2\right)(5x-2)} = \frac{-40}{\left(5(x+h)-2\right)(5x-2)}$$

3. Find all solutions to each equation:

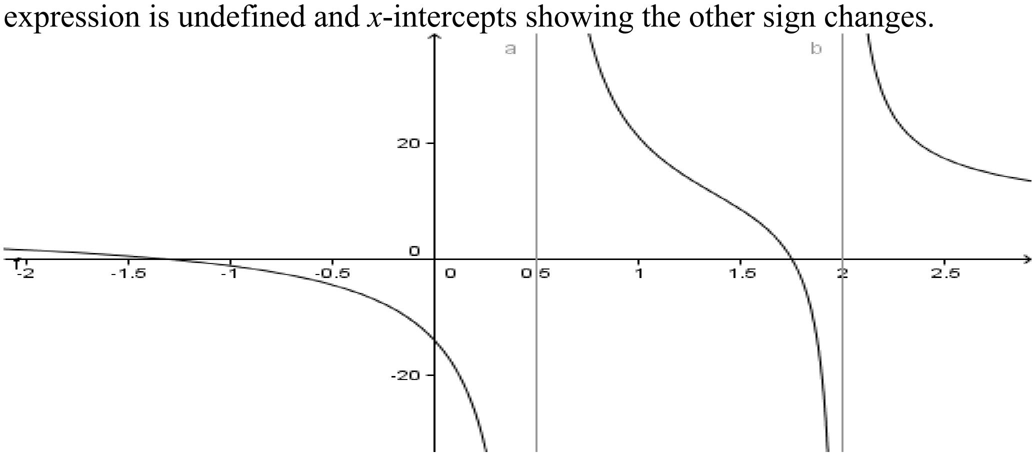
   a. $(x-1)^2 = \frac{4}{3} \Leftrightarrow x - 1 = \pm\frac{2}{\sqrt{3}} \Leftrightarrow \boxed{x = 1 \pm \frac{2\sqrt{3}}{3}}$

   b. $\frac{1}{x-1} - \frac{1}{x^2-1} = \frac{5}{4(x+1)} \Leftrightarrow 4(x+1) - 4 = 5(x-1) \Leftrightarrow \boxed{x = 5}$

4. Jamal has a collection of nickels, dimes and quarters, which, all together, weighs 93.9 grams. Dimes weigh 2.3 grams; nickels weigh 5.0 grams and the quarters, 5.7 grams. If the number of dimes is 3 more than the number of quarters, and the number of nickels is the number of quarters and dimes combined, how many quarters are there? Use the algebraic method to solve this problem.
SOLN: Let *N*, *D* and *Q* represent the numbers of nickels, dimes and quarters, respectively. Then $50N + 23D + 57Q = 939$, $D = Q+3$ and $N = D+Q$. Substituting, we have $N = 2Q+3$ and so
$50(2Q+3)+23(Q+3)+57Q = 180Q+219 = 939 \Leftrightarrow \boxed{Q=4}$ so 7 dimes, 11 nickels and 4 quarters.

5. The height of an isosceles triangle is three times the length of its base. If the area of the triangle is 12 square meters, what is the height of the triangle? Write your answer in simplest radical form and then approximate to the nearest hundredth of a meter.
SOLN: Area = half the product of base times height, and since the height is 3 times the base, the area is half of 3 times the square of the base. Setting this equal to 12 we have
$\frac{3}{2}b^2 = 12 \Leftrightarrow b^2 = 8 \Leftrightarrow b = 2\sqrt{2}$ so the height is $6\sqrt{2} \approx 6(1.414) \approx 8.48$ meters.

6. Solve the inequality $\frac{1}{x-2}+\frac{3x}{2x-1} \le \frac{9}{10}$. Write the solution using interval notation.
SOLN:
$$\frac{1}{x-2}+\frac{3x}{2x-1} \le \frac{9}{10} \Leftrightarrow \frac{20x-10+30x(x-2)-9(x-2)(2x-1)}{10(x-2)(2x-1)} \le 0 \Leftrightarrow$$
$$= \frac{12x^2+16x-21x-28}{10(x-2)(2x-1)} = \frac{4x(3x+4)-7(3x+4)}{10(x-2)(2x-1)} = \frac{(4x-7)(3x+4)}{10(x-2)(2x-1)} \le 0$$
Thus the zeros are, in increasing order, $-\frac{4}{3},\frac{1}{2},\frac{7}{4},2$ and since $x = 0$ is a solution to the inequality, we can conclude the solution set is $\left[-\frac{4}{3},\frac{1}{2}\right)\cup\left[\frac{7}{4},2\right)$ The graph of the left hand side of the last inequality is shown below, with vertical asymptotes a and b indicating the sign changes where the expression is undefined and *x*-intercepts showing the other sign changes.



7. Find an equation for the circle centered at (3,4) and passing through (0,0).
SOLN: $(x-3)^2+(y-4)^2 = 25$

8. Find an equation for the line passing through (0,0) and perpendicular to the line $2x+3y=5$.
SOLN: $y = \frac{3}{2}x$