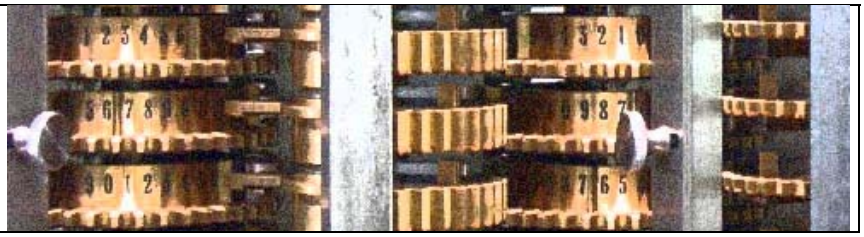
[Syllabus] [Calendar] [Compilers] [References]	

Physics 5 – Spring '11 – Assignment #2 - Bin2Dec

Study the following as part of this assignment

1. **Algorithm 1.2, *Conversion from Binary to Decimal***, and **Algorithm 1.3, *Conversion from Binary to Decimal by Horner's Method*** on page 5 of FCC++.
2. §2.9 *The Standard Library*, on pages 32 and 33 of FCC++, including the <cmath> file.
3. The `for` statement introduced in §3.8 on page 51 of FCC++.
4. §5.1 and §5.2 on pages 88 and 89 of FCC++, describing how to define, traverse and initialize an array. (Note that EXAMPLE 5.2 contains a syntax error. It should be `sqrt(10.0*i)`, with rounded parentheses, not square brackets. This is a function in the `<cmath>` file.)

Now, consider the C++ code below, which purports to implement Algorithm 1.2 in C++.

```
int main()
{
      double dec = 0.;
      // b will hold the binary number 10111011
      //(note that it's backwards with the least significant digit first)
      double b[8]={1.,1.,0.,1.,1.,1.,0.,1.};
      cout << "\ndec is initially = " << dec;
      for(int i=0;i<8;i++)
      {
            dec += b[i]*pow(2.,i);  // inefficient!
            cout << endl << "dec is increased by b[" << i << "]*"
                 << pow(2.,i) << " = " << b[i] << "*" << pow(2.,i)
                 << " to " << dec;
      }
      cout << "\n\nSo the binary number ";
      for (int i=7;i>=0;i--) cout << b[i];
      cout << " is " << dec << " in decimal " << endl;
}
```

This program has some debugging code built into it so you can see more of what is happening while it's happening. The output of this code is

```
dec is initially = 0
dec is increased by b[0]*1 = 1*1 to 1
dec is increased by b[1]*2 = 1*2 to 3
dec is increased by b[2]*4 = 0*4 to 3
dec is increased by b[3]*8 = 1*8 to 11
dec is increased by b[4]*16 = 1*16 to 27
dec is increased by b[5]*32 = 1*32 to 59
dec is increased by b[6]*64 = 0*64 to 59
dec is increased by b[7]*128 = 1*128 to 187

So the binary number 10111011 is 187 in decimal
```

Copy this code into your compiler and get it to work. Note that you'll need both the `<iostream>` and `<cmath>` files from the Standard Library. Try modifying parts of it to help understand how it works.

Next, consider the C++ code below, which is significantly improved, but not quite Horner's method.

```
int main()
{
      int dec = 0.;
      // b will hold the binary number 10111011
      //(note that it's backwards with the least significant digit first)
      int b[8]={1,1,0,1,1,1,0,1};
      int power_of_2 = 1;
      cout << "\ndec is initially = " << dec;
      for(int i=0;i<8;i++)
      {
            dec += b[i]*power_of_2;  // first multiplication
            cout << endl << "dec is increased by b[" << i << "]*"
                 << power_of_2 << " = " << b[i] << "*"
                 << power_of_2 << " to " << dec;
            power_of_2 *= 2;   // second multiplication
      }
      cout << "\n\nSo the binary number ";
      for (int i=7;i>=0;i--) cout << b[i];
      cout << " is " << dec << endl;
}
```

The output of this code is identical to that of the previous program (check this by modifying your program) but the program does not require the `<cmath>` file and doesn't use the pow() function which won't take the first argument as an int. Thus the program itself is much shorter (no `<cmath>` header, requires less memory (`int` types instead of `double`) and runs faster (multiplication is quicker than exponentiation).

Horner's method (algorithm 1.3) significantly improves on this by doing about half as many multiplications.

```
int main()
{
      // b will hold the binary number 10111011
      //(note that it's backwards with the least significant digit first)
      int b[8]={1,1,0,1,1,1,0,1};
      int dec = b[7];
      cout << "\ndec is initially = " << dec;
      for(int i=6;i>=0;i--)
      {
            cout << "\n2*" << dec << " + " << b[i] << " = ";
            dec = 2*dec+b[i];  //Horner's method
            cout << dec;
      }
      cout << "\n\nSo the binary number ";
      for (int i=7;i>=0;i--) cout << b[i];
      cout << " is " << dec << endl;
}
```

The output of this code is

```
dec is initially = 1
2*1 + 0 = 2
2*2 + 1 = 5
2*5 + 1 = 11
2*11 + 1 = 23
2*23 + 0 = 46
2*46 + 1 = 93
2*93 + 1 = 187

So the binary number 10111011 is 187 in decimal form
```

Your assignment, should you choose to accept it, is to write a C++ program that will prompt the user for a number less than 256, convert that number to a binary number which is stored in an array of 8 digits, display that number and then use Horner's method to convert it back into decimal. There is a WikiHow on converting a decimal to binary which may help. The output of your program should look something like this:

```
Please enter a non-negative integer less than 256:
187

The decimal equivalent of 187 is 10111011

Horner's method converts this back to 187
```

As discussed in class, Horner's algorithm extends to evaluating more general polynomial forms. To see this, note that Horner's form of the binary representation of 187:

$$\begin{aligned}
10111011_2 &= 1\times 2^7 + 0\times 2^6 + 1\times 2^5 + 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2 + 1 \\
&= \left(1\times 2^6 + 0\times 2^5 + 1\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2 + 1\right)2 + 1 \\
&= \left(\left(1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2 + 0\right)2 + 1\right)2 + 1 \\
&= \left(\left(\left(1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2 + 1\right)2 + 0\right)2 + 1\right)2 + 1 \\
&= \left(\left(\left(\left(1\times 2^3 + 0\times 2^2 + 1\times 2 + 1\right)2 + 1\right)2 + 0\right)2 + 1\right)2 + 1 \\
&= \left(\left(\left(\left(\left(1\times 2^2 + 0\times 2 + 1\right)2 + 1\right)2 + 1\right)2 + 0\right)2 + 1\right)2 + 1 \\
&= \left(\left(\left(\left(\left(\left(1\times 2 + 0\right)2 + 1\right)2 + 1\right)2 + 1\right)2 + 0\right)2 + 1\right)2 + 1
\end{aligned}$$

Can be generalized to an arbitrary seventh degree polynomial:

$$\begin{aligned}
p(x) &= a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \\
&= \left(\left(\left(\left(\left(\left(a_7x + a_6\right)x + a_5\right)x + a_4\right)x + a_3\right)x + a_2\right)x + a_1\right)x + a_0
\end{aligned}$$

So the algorithm suggested by this is to start with the leading term a_n and then repeatedly (1) multiply by x and (2) add the next a_{n-1}.

To reverse this, (1) put the remainder after division by x in the next significant digit and (2) divide by x.