What I Learnt at the Joint Meetings of the AMS/MAA

-by Geoff Hagopian, 1/07

Due to Delta (airline) operators rescheduling my flight from Atlanta, I was delayed about 4 hours and so missed Friday's events entirely. This meant getting my schedule early Saturday http://www.ams.org/notices/200701/neworleans-prog.pdf morning and spending part of each 20 minute (or so) session trying to figure out where to go next. There were about 8 talk rooms per floor at the 4 floors at the Sheraton and more at the Marriott, so that meant lots of dashing across Canal Street.

There were several excellent threads for armchair math teachers like me. And many that speak directly to ways to make teaching our curriculum more better fun. I started in the *Research on Teaching and Learning* thread.

Martin Flashman (CS Humboldt) gave a talk *What Place Does Philosophy Have in Teaching Mathematics*? Gödel's Theorem is typical of the sort of mathematical breakthroughs which deserve more attention at the undergraduate level. Flashman's web site http://www.humboldt.edu/~mef2/logicsites.html indicates extensive research in this area, including a link to a proof of Gödel's theorem, http://www.apronus.com/math/goedel.htm . The talk was more about the ontological and epistemological aspects of learning and knowing numbers, points, lines, etc.

More good discussion about proofs at http://www.cut-the-knot.org/proofs/index.shtml.

Bret Benesh, Harvard, reported on his study of *How Your Students Use Their Textbook*, http://www.pmena.org/2006/cd/SOCIO-CULTURAL ISSUES/SOCIO-CULTURAL ISSUES-0015.pdf finding, without amazement, that students tend to focus on examples and key equations and skip the words.

M. Axtell, Wabash College, found that if students are asked to submit answers to short questions about the reading just before the start of class, they typically copy definitions but do not remember them and are more likely to retain conceptual and computational information.

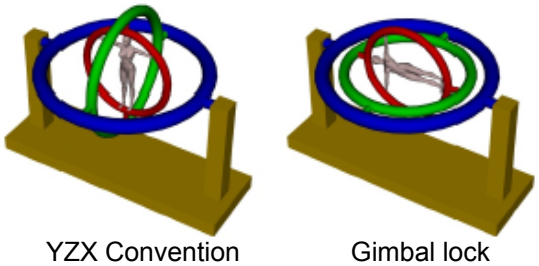
Vilma Mesa, U. Mich, contrasted the ways in which 12 calculus texts (Adams, 2003; Apostol, 1967; Goldstein ***et al.***, 2004; Hughes-Hallett ***et al.***, 2005; Larson ***et al.***, 2006; MacCluer, 2006; Ostebee & Zorn, 2002; Simmons, 1996; R. T. Smith & Minton, 2002; Stewart, 2003; Thomas ***et al.***, 2001; Zenor ***et al.***, 1999) present initial value problems and concludes that some texts are better at encouraging students to check the reasonableness of their solutions and engage in the "look back" stage of problem solving that may improve students' metacognitive and proving skills.

At this point I moved to the thread on *Innovative and Effective Ways to Teach Linear Algebra.*

Russell Blyth, St. Luis U., http://sylow.slu.edu/ talked on *Pedagogy and Visualization: two Aspects of the Use of a CAS in Linear Algebra*.

The parameters of pitch (angle up/down), yaw (angle left/right on a vertical axis) and roll (angle left/right on the horizontal axis) were ardently exhibited on the flight from Palm Springs to Atlanta. There were some wicked winds that made motion much more…diverse. I survived the flight to hear Helmer Aslaksen, U. of Singapore, http://www.math.nus.edu.sg/aslaksen/ give a nice talk demonstrating how to work with both local and global rotations and how a weird phenomenon (**Gimbal lock**) occurs when components of the local and global reference frames coincide. For instance, rotating 90 deg about an axis can make another axis in the local frame coincide with the a third axis in the global frame and then you have only two axes for rotation. This phenonmenon has occurred on the MIR spacecraft and caused system failures.

Aslaksen explained that he wouldn't use the nude figure for the classroom – but methinks this is art, so why not have a nude?



YZX Convention Gimbal lock

Aslaksen used the free software available at https://www.daz3d.com/ to visualize how Gimbal lock occurs. The algebra of rotation matrix multiplications shows the mathematical implications nicely. Tom Hanks had to pretend he understood this in Apollo 13, the movie.

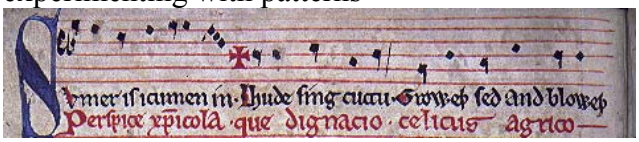
Stephen Hilbert, Ithaca College, gave a short discussion of how he uses a motivational Markov Matrix model to get students to examine how intial values can affect solutions of an iterated matrix system. He has students use Matlab, but only on an as needed basis: "from here to here, here's what I would do [in Matlab] and I can iterate it." Decomposing the initial value vector relative to any basis can be shown to produce results that depend on the eigensystem parameters.

Petre Ion Ghenciu, University of Wisconsin-Stout, described his fledgling experiences teaching with "The Discovery Method" http://www.discovery.utexas.edu/rlm/reference/dancis_davidson.html a la R. L. Moore in glowing terms. http://www.discovery.utexas.edu/rlm/reference/dancis_davidson.html This was the first among many speakers I heard extolling the virtues of this method. The very next speaker, Stephen B. Maurer, Swarthmore, http://www.swarthmore.edu/NatSci/smaurer1/ was also a great proponent. He said about a fifth of Swarthmore freshmen have a 5 on the Caclulus AB AP exam and they give them this "discovery" treatment. It's a small seminar class whose materials are available here. https://blackboard.swarthmore.edu/webapps/portal/frameset.jsp?tab=courses&url=/bin/common/course.pl?course_id=_24445_1 Most telling is his problem set http://www.swarthmore.edu/NatSci/smaurer1/Math28P/Problems/allF05probs.pdf which serves as both a text and the cornerstone "extensive syllabus" that the Moore method requires. http://www.discovery.utexas.edu/rlm/reference/quick_start-3.pdf

After lunch I sampled the *Mathematical Techniques in Musical Analysis* thread, beginning with *Yea, Why Try Her Raw Wet Hat*. By Robin J. Wilson, Open University, UK, http://www.gresham.ac.uk/event.asp?PageId=39&EventId=462 who talked about how composers use mathematical ideas in their compositions. Composers often use magic squares, alternate logarithmic scales, translation (fugue), and other constructions borrowed from mathematics to compose. From Xenakis' Metastaseis http://en.wikipedia.org/wiki/Metastasis_(Xenakis_composition) to Sir Peter Maxwell Davies' Mirror of Whitening Light to Carlton Gamer's Organum, a 31-tone rendering of a snippet from Ezra Pound's Canto LXXXI http://www.uncg.edu/eng/pound/canto.htm

What thou lovest well remains,
the rest is dross
What thou lov'st well shall not be reft from thee
What thou lov'st well is thy true heritage
Whose world, or mine or theirs
or is it of none?
First came the seen, then thus the palpable
Elysium, though it were in the halls of hell,
What thou lovest well is thy true heritage
What thou lov'st well shall not be reft from thee

Tallis' canon, *Sumer Is Icumen In* (note that on Wiki translation is a parody by…none other than Ezra Pound again) http://en.wikipedia.org/wiki/Sumer_Is_Icumen_In#Music is a classic example of an early composer experimenting with patterns

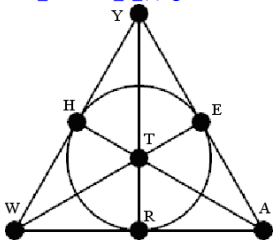


Wilson claims that we name seven colors (roygbiv) in the rainbow as a result of Newton's combined researches into optics and music.

The symmetry of Bach's palindromic Crab Canon http://www-personal.umich.edu/~msmiller/sounds/s1079.3.midi is the model for discussion in Penrose's Godel, Escher, Bach. http://www.evl.uic.edu/swami/crabcanon The Menuet from Haydn Sonata #41 http://www.kunstderfuge.com/_/sonata_41_2_(c)iscenko.mid is another good palindromic example.

Bartok's *Mikrokosmos* is an example of a composition using inversional symmetry. http://www.kunstderfuge.com/_/bartok_mikrokosmos_selection_1_(c)oguri.mid

The title of Robin's talk was based on a geometrical construction called the Fano plane, which is a projective plane consisting of the 7 points W, H, Y, E, A, T, R and



the 7 'lines' YEA, WHY, TRY, HER, RAW, WET and HAT. Wilson describes how this diagram is used to group instruments but is consipicuously vague about the details.

Taking the escalator up 2 floors at the Sheraton I found the thread *Reconceptualizing Content Courses for Prospective High School Mathematics Teachers* where Neil Portnoy, et al were talking on *Connecting Postsecondary and Secondary Mathematics*. The claim is that using the same curriculum for people who want to do mathematics and people who want **to** teach it isn't necessarily the ideal world. They advocate a book *Ways to Think About Mathematics Activities and Investigations for Grade 6-12 Teachers* that they co-author. http://www.corwinpress.com/booksProdSampleMaterials.nav?prodId=Book226041 Like the Moore method, it's very problem-driven.

Tanya Cofer http://www.neiu.edu/~tcofer/ described her study interviewing k-12 teachers about fundamental issues of abstract algebra encountered at the most basic levels. "Why is division by zero undefined?" she asks, and then classifies responses as belonging to categories such as AMA: "Abstract Math Analysis" (the correct approach) or A: "Analogy" and R: "Rule." Responses she's recorded are often complex – jumping from one type of justification to another and even self-contradictory, at times. The meaning of 0/0 causes even more diversity in responses. Many problems arise from thinking of "nothing" as a "something," or not. A respondent's metaphor of apples per basket was especially amusing.

In the *Teaching Innovations in Real Analysis* thread David Scott, U. Puget Sound, spoke on how it might be more better to simply ask students to find delta as a function of epsilon rather than show that, given any epsilon we can find a delta.

I skipped sessions again to *Communication Theory in Undergraduate Courses* to catch Sarah Spence Adams, Olin College of Engineering, talk on *A Motivational Course in Cryptology and Coding Theory*, http://faculty.olin.edu/~sadams/CCT/index.html where she described a course they've developed in hopes of grabbing smart kids for non-math majors.

Down on the 2nd floor Nathaniel Miller, U. of No. Colorado, http://hopper.unco.edu/faculty/personal/miller/index.html was speaking on The Philosophical Status of Diagrams in Euclidean Geometry. In *Euclid and His Twentieth Century Rivals: Diagrams in the Logic of Euclidean Geometry*, http://www.mathsci.unco.edu/faculty/personal/miller/diagrams/ Miller writes "Euclid's proofs in particular, have mostly fallen out of the standard mathematics curriculum. This is at least in part because Euclid's *Elements*, http://www.sunsite.ubc.ca/DigitalMathArchive/Euclid/byrne.html which was viewed for most of its existence as being the gold standard of careful reasoning and mathematical rigor, has come to be viewed as being inherently and unsalvageably informal and unrigorous." As the diagram (from http://hopper.unco.edu/faculty/personal/miller/diagrams/overview.pdf) below suggests, Miller

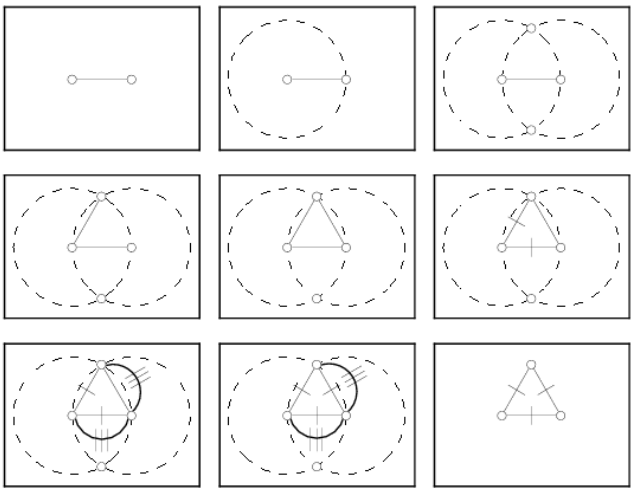


Fig. 1. Euclid's first proposition.

claims that "We can give a formal definition of which arrangements of lines, points, and circles in the plane are represented by a given diagram; this is the semantics. Finally, we can give precise rules for manipulating the diagrams---rules of construction and inference. All of these rules can be made entirely precise and implemented on a computer."

Back up in the *Reconceptualizing Content Courses of Prospective High School Math Teachers* thread Greisy Winicki-Landman, Cal Poly Pomona, advocated a provocative approach:

> Instead of asking students "Show that there is no...." I generally ask my students "Find a..." without giving any additional information. The reader may consider I lied to them... I don't. With the task formulated affirmatively, the solver will try to accomplish the task and only after same attempts (sometimes many attempts or even never...) he will suspect that the task is "very hard", or almost "impossible". Then, he is in front of a metamathematical decision: to keep on trying to do what he was asked to do or to be "insolent" and change direction trying to prove that he had been asked to do something that is not possibly done. From an educational point of view, I think it is very important to have mathematics students discuss the role of impossible things in mathematics

Tim McDevitt, Elizabethtown College, http://www.etown.edu/Directory.aspx?faculty=942b0437-609e-475c-b5f7-c458cb639ad5 gave a presentation in the *Communications Theory* thread on *Making Elliptic Curves Accessible to Undergraduates*. There were some amazing Mathematica animations used to illustrate how the rational points are connected.

I stopped by the WebAssign http://www.webassign.net/ which is taking over operations for ThompsonNow. I'm familiar with WebAssign through my kid's assignments at Palm Desert High School.

Sunday morning could have started with Lee Lindblom, Cal Tech, http://www.physics.umd.edu/grt/people/lindblom.html and his address *Generalized Harmonic Evolutions of Binary Black Hole Spacetimes* in the *Numerical Relativity* thread, but you get lots of his similar articles here: http://www.slac.stanford.edu/spires/index.shtml/hep .

It may instead have observed Michael B. Ward, Western Oregon U., http://www.wou.edu/~wardm/ward.html whose web site has a link to the nifty Geometry Explorer (a GSP-like feeware), http://homepages.gac.edu/~hvidsten/gex/download.html speak on Cayley-Sudoku Tables. A **Latin square** of order n is an $n \times n$ array containing the symbols 1, . . . , n such that each symbol occurs once in each row and once in each column.

The Cayley table of a group is a Latin square. In fact, the Cayley table of a binary system $(A, \circ)$ is a Latin square if and only if $(A, \circ)$ is a quasigroup (left and right division are uniquely defined, i.e. the equations $a \circ x = b$ and $y \circ a = b$ have unique solutions x and y for any a and b.)

For example,

$\circ$	a	b	c
a	b	a	c
b	a	c	b
c	c	b	a

…or one with an identity element:

$\circ$	e	σ	τ	α	β	γ
e	e	σ	τ	α	β	γ
σ	σ	τ	e	β	γ	α
τ	τ	e	σ	γ	α	β
α	α	γ	β	e	τ	σ
β	β	α	γ	σ	e	τ
γ	γ	β	α	τ	σ	e

Izabela Kanaana http://www.sonoma.edu/users/k/kanaana/ and Bala Ravi-Kumar, Sonoma State U., talked on the *Row-Filled Completion Problem for Sudoku Latin Squares*. http://www.eddaardvark.co.uk/sudokusolver.html

Philip A. Cobb, Queensborough Community College, talked on algorithms for solving Sudoku such as this check for an "only position" situation. I've been looking through this code to see how it fits in some context. I gather it's treating the sudoku as a 9x9 array and then grabbing a row into a string of size 9?

```
set = "";
for (cx=1;cx<=9;cx++){
  set=set+cset[r][cx];
} // holds set nums
for (n=1;n<=9;n++) {
  sn = String(n);
  if (set.indexOf(sn)==-1)
  // n not set
  {
    count = 0;
    for (i=1;i<=9;i++) {
    //count if possible
      if (cposs[r][i].indexOf(sn)>-1)
        {fpos = i; count++;  }
    }
    if (count==1) {
      cset[r][fpos]=sn;
      namec = "("+r+","+fpos+")";
      _root[namec].cellvalue = sn;
      cposs[r][fpos] = "";
    }
  } //ends n not assigned
} //end loop through numbers 1 to 9
```

At this point it was time for the two-hour second half of the mini-course I signed up for on the *Mathematics and Geometry of Voting*, by Donald G. Saari, UCI. http://www.math.uci.edu/~dsaari/ He's got a humorous account of trying to explain voting to a fourth grade class. http://www.math.uci.edu/~dsaari/fourthgrade.pdf Saari was the cover story for the March, 2005 http://www.math.uci.edu/~dsaari/bioinfo.htm edition of the College Math Journal – a very humorous bio. It seems that Don has been making great strides forward in social choice theory. I'm looking forward to using some of his ideas in the Math 13 courses I'm teaching next semester.

In *The Other Book Nobody Read: George Rheticus and the Opus Palatinum*, of the *History of Math* thread, Glen R. Van Brummelen, Quest U., http://faculty.bennington.edu/~gvanbrum/ described the *Opus palatinum de triangulis* of Rheticus, http://en.wikipedia.org/wiki/Georg_Joachim_Rheticus the (one and only) student of Copernicus. http://en.wikipedia.org/wiki/Copernicus He claims Rheticus was probably the first to define trigonometric functions directly in terms of right triangles instead of circles, with tables for all six trigonometric functions; this work was finished by Rheticus' student Valentin Otho in 1596. In any case, the most amazing tables of trig functions are given in this huge book and the methods used to produce them are of particular interest. Mostly additive, half angle and complementary identities involving angles of 18, 30, 36 and 40 degrees. Also, the approximation $\sin x \sim x$ if $|x|<<1$. Apparently Rheticus was aware of one of my favorites: Al Khashi, who use the identity $\sin 3x = 3\sin x - 4\sin^3 x$ to solve for $\sin 1^\circ$ in terms of $\sin 3^\circ = \sin\left(\frac{1}{2}\left(36^\circ - 30^\circ\right)\right)$. All values in the table are multiplied by 1010 and Rheticus introduces the sec, csc, cot ratios for the first time but never uses sine verbiage in the expectation that it wouldn't survive in modern usage.

Mark Krebs, Cal State, LA, http://www.calstatela.edu/faculty/mkrebs/research/research.html spoke on sudoku puzzles. http://www.calstatela.edu/faculty/gbrookf/pubs/beaucoupsudoku.pdf Especially 4x4 puzzles and Robin Blankenship, Morehead State University spoke on the number of leaves you would need to draw a non-interesecting graph on a 4x4 grid of nodes where each row and each column is a complete graph.

Denny Gulick presented *Development Mathematics Program at the University of Maryland* in the *Contributed Papers* section. This remediation program was originally presented in the 12/03 edition of Focus http://www.maa.org/pubs/dec03.pdf and I remember discussing it with Thang at the time. In an attempted to reduce a semester of developmental math without compromising the quality of their program, they've completely revamped their system so that all entering students are given a modified version of the MAA placement exam http://www.maplesoft.com/maa/?loc=product&P=MAA010606 and then the lowest 40% are told to go and take the full remedial program which is self-paced on a computer platform and awarded pass/fail based on their work on tests, homework, attendance (6hr/wk) and success in computer modules. The

top 60% are placed into a 5-week intensified form of the remedial class after which can transfer into a 11 week intensified version of the next course, if successful – if not, they're placed back in the regular remedial course already in progress.

Denny reported great success in this program over the last 5 years. 85% of the 60% (about half the total) are successful in eliminating the extra semester and the other half don't necessarily lose a semester in the bargain.

Michelle Dedeo, U. North Florida, http://www.unf.edu/~mdedeo/ described her experiences revamping the college algebra course there. Her efforts to make the course more consistent and engaging for students sound very familiar. She tried various softwares for organizing homework and tests and ended up with the Hawkes System http://www.hawkeslearning.com/PC_COL.htm – after rejecting MyMath among others. She was quite enthusiastic about the quality and success of this program at UNF and I think it's worth checking out. It's based on a mastery system and is keyed into a text that looks quite appropriate to what we're trying to do with this precalc orphan.

I bought a few books at the conference. I got Nobuyuki Yoshigahara's Puzzles 101 http://www.maa.org/editorial/mathgames/mathgames_06_28_04.html which has nice open-ended problems like:

> There are six edges connecting any two vertices of a square and they have two lengths, either 1 (edge) or sqrt(2) (diagonal.) Can you rearrange the vertices so they're in a plane and there are still only two lengths connecting any two points? How many different arrangements can you think of? No two points are allowed to be in the same location.

Peter Winkler's http://www.math.dartmouth.edu/~pw/ *Mathematical Puzzles, A Connousseur's Collection* looks fabulous. This one is from automata theory, "Life" like:

An infection spreads among the squares of an $n \times n$ checkerboard in the following manner: If a square has two or more infected neighbors, then it becomes infected itself (only the four orthogonal neighbors count.) Prove that, if you begin with fewer than n infected squares then there is no initial arrangement that will lead to all squares being infected.

As for Life, http://en.wikipedia.org/wiki/Conway's_Game_of_Life I also got John Conway's *On Numbers and Games*. The book builds the number system in ways inspired by Dedekind and Cantor and culminating in what Knuth called the surreal numbers in a way that is somehow analogous to a game played between Left and Right ("our sympathies are usually with Left.") Follow the Conway link http://www.math.princeton.edu/facultypapers/Conway/ to see his very charming lectures on all kinds of neat stuff.

I was disappointed to see John Derbyshire's comments on his visit to New Orleans http://article.nationalreview.com/?q=OGYzYzI5NzU3ZmEyYWEwYjg5MjBiODdmN2ZmMGE1ZDg in the National Review. Now, I read and enjoyed his book, *Prime Obsession*, http://www.olimu.com/Riemann/Riemann.htm and even based my last Science/Math Lecture Series talk on it, http://faculty.collegeofthedesert.edu/ghagopian/RiemannHypothesis.htm but I've since read Marcus Du Sautoy's *Music of the Primes*, and find it, overall, a better read. http://www.musicoftheprimes.com/

I didn't know Derbyshire is one of these weird blue-blood Brit conservative-come-neocon types, but his comments about N.O. reveal him to be uneasy about race: "I think this is the blackest American city I have been in," but quite willing to spout off about things he knows little about: "The Mardi Gras thing goes back forever (well, to 1699), and it's been camp, camp, camp all the way." Typical of these intellectual neocons, he's agile with the rhetoric of inoculating oneself against the evil one intends to do before doing it: "It is, of course, grossly unfair to pass any kind of judgment on a city after a two-day visit," he says, and then proceeds to do just that. This is especially egregious since he describes spending much of his time at a tourist's mall. Sheesh.