References

1. R. H. Bruck, A Survey of Binary Systems, Springer-Verlag, 1958.
2. I. M. James, On H-spaces and their homotopy groups, Quart. J. Math. Oxford Ser. (2), 11 (1960) 161–179.

NEW ANGLES ON AN OLD GAME

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A standard calculus exercise shows that the trajectory of a projectile, subject only to gravity, is a parabola. If we think of the projectile as a basketball, two additional interesting questions can be asked.

1. What is the smallest initial angle at which a basket can be made?
2. What is the minimum initial velocity needed to make a basket and what angle corresponds to this velocity?

The basketball problem differs from the problem of a bullet hitting a target in one significant detail. The bullet can hit the target either while the bullet is rising or while it is falling. The basketball must go through the hoop while the ball is falling.

We assume that both the ball and the basket are points and that the motion is two-dimensional. The ball is shot from $(0,0)$ at time $t = 0$ with an initial velocity, v_0, and angle, θ. The basket is located at (l, h), where both l and h are positive.

As usual, one easily derives that, at time t, the position of the ball is given by

$$x(t) = v_0(\cos\theta)t \qquad y(t) = -\tfrac{1}{2}gt^2 + v_0(\sin\theta)t$$

where g is the gravitational constant.

A basket is made if there is t_0 such that

(1) $x(t_0) = l$ and $y(t_0) = h$, and

(2) $y'(t_0) < 0$.

Upon our using the equations of motion, these conditions become, respectively,

$$v_0^2 = \left(\tfrac{1}{2}gl^2\sec^2\theta\right)/\,(l\tan\theta - h) \tag{1}$$

and

$$v_0^2 < gl/\,(\sin\theta\cos\theta), \tag{2}$$

which together imply:

THEOREM 1. *For a basket to be made, the initial angle must be greater than* $\operatorname{Tan}^{-1}(2h/l)$.

By substituting $\tan^2\theta + 1$ for $\sec^2\theta$, equation (1) can be transformed into an equation quadratic in $\tan\theta$. This equation has a real solution for $\tan\theta$ when

$$v_0^4 - 2hgv_0^2 - g^2l^2 \geqslant 0.$$

This holds when

$$v_0^2 \geqslant g\left(h + (h^2 + l^2)^{1/2}\right). \tag{3}$$

Thus the minimal initial velocity at which a basket can be made (i.e., for which $\tan\theta$ has a real solution) must satisfy equality in (3). At this velocity

$$\tan\theta = \left(h + (h^2 + l^2)^{1/2}\right)/l. \tag{4}$$

THEOREM 2. *To make a basket with the minimal initial velocity, the initial angle should be*

$$\tfrac{1}{2}(\operatorname{Tan}^{-1}(h/l) + (\pi/2))$$

(*i.e., the minimal velocity is required at the angle halfway between the line-of-sight and the vertical*).

Proof. We first note that

$$\left(h + (h^2 + l^2)^{1/2}\right)/l > 2h/l;$$

so, by Theorem 1, the basket can be made at the angle θ of equation (4). It thus suffices to show that

$$\tan\left(\tfrac{1}{2}(\mathrm{Tan}^{-1}(h/l) + (\pi/2))\right) = \left(h + (h^2 + l^2)^{1/2}\right)/l.$$

This follows immediately from Fig. 1 by noting that $|AC| = |DC|$ and therefore $\angle DAC = \angle DAE$.

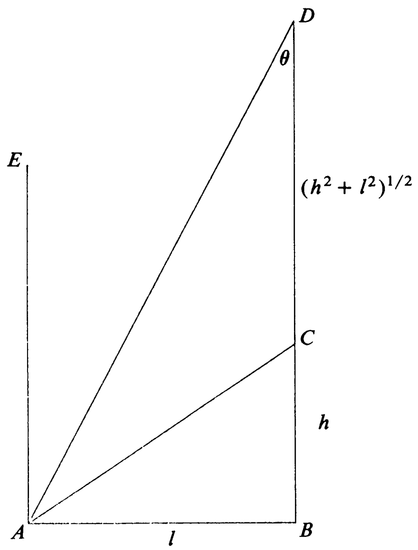

FIG. 1

MATHEMATICAL EDUCATION

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TRANSLATION DIFFICULTIES IN LEARNING MATHEMATICS

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Recent mathematics textbooks have increasingly emphasized applications. Mathematical modeling is a critical component of applications, as Rubin [8] points out. Unfortunately, results