Now differentiate each of the relationships

$$\int \cos(px)\,dx = p^{-1}\sin(px) + C, \qquad (p \neq 0) \qquad (8a)$$

$$\int \sin(px)\,dx = -p^{-1}\cos(px) + C, \qquad (8b)$$

n times with respect to p and use (7), with (1) and (2), in order to obtain

$$\int x^n \cos\left(px + \frac{n\pi}{2}\right) dx = n!\, p^{-(n+1)} \sum_{i=0}^{n} \frac{(-1)^{n-i}(px)^i}{i!} \sin\left(px + \frac{\pi i}{2}\right) + C, \qquad (9a)$$

$$\int x^n \sin\left(px + \frac{n\pi}{2}\right) dx = n!\, p^{-(n+1)} \sum_{i=0}^{n} \frac{(-1)^{n-i+1}(px)^i}{i!} \cos\left(px + \frac{\pi i}{2}\right) + C. \qquad (9b)$$

The right-hand side of (9a), (9b) followed by taking $v = p^{-1}$ (instead of taking $u = p^{-1}$, as in Examples 1, 2).

If $n = 2m + 1$, then

$$\cos\left(px + \frac{n\pi}{2}\right) = (-1)^{m+1}\sin(px)$$

and

$$\sin\left(px + \frac{n\pi}{2}\right) = (-1)^{m}\cos(px).$$

If $n = 2m$, then

$$\cos\left(px + \frac{n\pi}{2}\right) = (-1)^{m}\cos(px)$$

and

$$\sin\left(px + \frac{n\pi}{2}\right) = (-1)^{m}\sin(px).$$

Thus, $\int x^n \cos(px)\,dx$ and $\int x^n \sin(px)\,dx$ are readily obtained from (9a), (9b). And, the substitution $t = \text{Arcsin}\,x$ and $t = \text{Arccos}\,x$ reduce $\int(\text{Arcsin}\,x)^n\,dx$ and $\int(\text{Arccos}\,x)^n\,dx$ to $\int t^n \cos t\,dt$ and $-\int t^n \sin t\,dt$, respectively.

Our approach may be viewed as a useful supplement (not a replacement) to the method of integration by parts. Under certain conditions, it can be effectively applied to some important classes of proper and improper integrals.

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How to Make a Bank Shot

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In Figure 1, a ball at C is to be banked off side BD in order to land in pocket A. Ordinarily, Heron's Reflection Principle could be used to locate the desired impact point Q: If C is reflected about BD to a point E and line AE is drawn, then Q is the point of intersection of BD with AE. This follows since $\sphericalangle AQB$ and $\sphericalangle CQD$ both equal $\sphericalangle DQE$.

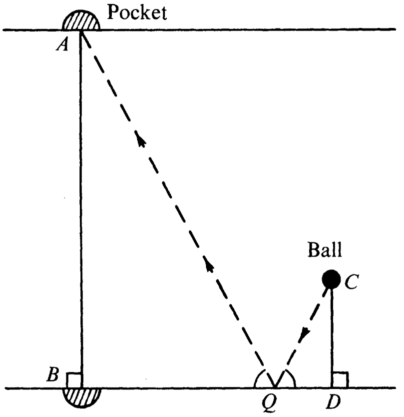

Figure 1.

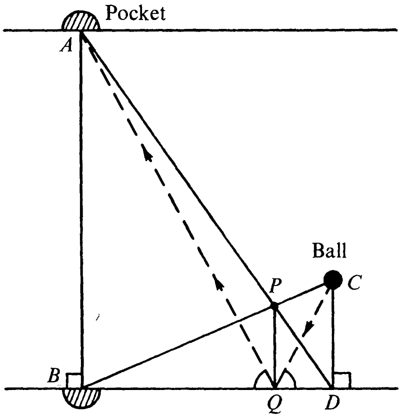

Figure 2.

Since point E is not on the pool table, this method may entail some practical difficulties. Students who are "on the ball," however, might consider Figure 2, in which $PQ \perp BD$. Clearly,

$$\frac{CD}{BD} = \frac{PQ}{BQ} \quad \text{and} \quad \frac{AB}{BD} = \frac{PQ}{QD}.$$

Thus, $AB/CD = BQ/DQ$ and triangles ABQ and CDQ are similar. Accordingly, $\sphericalangle AQB$ equals $\sphericalangle CQD$.

Interfractile Ranges

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Let $\{x_1 \leqslant x_2 \leqslant \cdots \leqslant x_n\}$ be a set of real numbers with mean μ and standard deviation σ. Recently [TYCMJ 13, November 1982, p. 322] Page and Murty established that

$$\mu - \sigma\sqrt{\frac{n-r}{r}} \leqslant x_r \leqslant x_{n-r+1} \leqslant \mu + \sigma\sqrt{\frac{n-r}{r}} \qquad \text{for} \quad r = 1, 2, \ldots, \left[\frac{n+1}{2}\right]. \tag{*}$$

(Here $[\alpha]$ denotes the greatest integer less than or equal to α.) Our objective is to illustrate how (*) can be used to obtain bounds on some other important measures of dispersion.