

Write all responses on separate paper. Do not use a calculator. Show your work for credit.

1. Joe has two types of bolts. A group of thirty of the longer bolt and eighteen of the shorter bolts weighs 300 ounces. A group of twenty five of the longer bolts with fifteen of the shorter bolts weighs 300 ounces. How much does a longer blot weigh?
2. A 17% alcohol mixture is combined with a 71% alcohol mixture to produce 50 liters of 32% alcohol mixture. How much of each ingredient is used?

3. Write an equation of the line with slope 3% passing through the point $\left(\frac{1}{2}, \frac{8}{5}\right)$.

Express your answer in the slope-intercept form: $y = mx + b$.

4. Write an equation of the line passing through the points $\left(\frac{3}{2}, \frac{5}{7}\right)$ and $\left(\frac{5}{4}, \frac{1}{14}\right)$.

Express your answer in the slope-intercept form: $y = mx + b$.

5. Write an equation for the line parallel to the line whose equation is $5y - 4x = 17$.
Express your answer in the slope-intercept form: $y = mx + b$.

6. Use the graphical method to solve the system of equations for x and y :

$$4x + 3y = 24$$

$$4x - 3y = 0$$

7. Write the equation of the horizontal line passing through $(7.2, -4.3)$. What are the coordinates of the point where this line intersects the line $2x + 3y = 60$?

8. Solve the system of equations by graphing:

$\frac{x}{3} + \frac{y}{4} = 6$
$2y = x + 4$

9. Write a system of equations that is
 - a. dependent.
 - b. inconsistent.

10. Solve the system of equations either by graphing, elimination or substitution. State whether the system is independent, dependent or inconsistent.

$$5x - y = 8$$

$$3x + 2y = 30$$

Math 54 – Chapters 5 and 6 Test Solutions – Spring'10

1. Joe has two types of bolts. A group of thirty of the longer bolt and eighteen of the shorter bolts weighs 300 ounces. A group of twenty five of the longer bolts with fifteen of the shorter bolts weighs 250 ounces. How much does a longer bolt weigh?

SOLN: It's hard to tell! Let L = the weight of a longer bolts in ounces and S = the weight of a shorter bolt. Then the two equations you can set up based on this information are equivalent:

$$30L + 18S = 300 \Leftrightarrow 25L + 15S = 250 \Leftrightarrow 5L + 3S = 50$$

However, the solutions are limited by the requirement that L and S have natural number values.

Since the right side is a multiple of 5, so must the left side be a multiple of 5. So S must be a multiple of 5. If $S = 5$ then $L = 7$ is the only natural number solution where the longer bolts weight more than the shorter bolts, which makes sense. It could also be that $S = 10$, $L = 4$ or $S = 15$, $L = 1$, but then the shorter bolts would weight more.

2. A 17% alcohol mixture is combined with a 71% alcohol mixture to produce 50 liters of 32% alcohol mixture. How much of each ingredient is used?

SOLN: Let x = the amount of 17% solution in the mixture. Then $50 - x$ is the amount of 71% alcohol solution in the mixture. Now the pure alcohol in the mixture must come from one of these ingredients and must be 32% of 50 = 16 liters. Thus $0.17x + 0.71(50 - x) = 16$ Expanding and collecting terms on the left side we have the equivalent equation

$35.5 - 0.54x = 16$ so that $0.54x = 19.5$ whence $x = 1950/54 = 325/9$ liters of 17% solutions and so there must be $125/9$ liters of the 71% solution.

3. Write an equation of the line with slope 3% passing through the point $\left(\frac{1}{2}, \frac{8}{5}\right)$.

Express your answer in the slope-intercept form: $y = mx + b$.

SOLN: Plugging into the point-slope form: $y - y_1 = m(x - x_1)$ we have

$$y - \frac{8}{5} = 0.03\left(x - \frac{1}{2}\right) \Leftrightarrow y = \frac{3}{100}x - \frac{3}{200} + \frac{320}{200} \Leftrightarrow y = \frac{3}{100}x + \frac{317}{200}$$

4. Write an equation of the line passing through the points $\left(\frac{3}{2}, \frac{5}{7}\right)$ and $\left(\frac{5}{4}, \frac{1}{14}\right)$.

Express your answer in the slope-intercept form: $y = mx + b$.

SOLN: Plugging into the slope formula, $m = \frac{\frac{5}{7} - \frac{1}{14}}{\frac{3}{2} - \frac{5}{4}} = \frac{\frac{10}{14} - \frac{1}{14}}{\frac{6}{4} - \frac{5}{4}} = \frac{9}{14} \left(\frac{4}{1}\right) = \frac{18}{7}$. Then plug into the

point-slope form: $y - y_1 = m(x - x_1)$ we have

$$y - \frac{5}{7} = \frac{18}{7}\left(x - \frac{3}{2}\right) \Leftrightarrow y = \frac{18}{7}x - \frac{27}{7} + \frac{5}{7} \Leftrightarrow y = \frac{18}{7}x - \frac{22}{7}$$

5. Write an equation for the line parallel to the line whose equation is $5y - 4x = 17$.

Express your answer in the slope-intercept form: $y = mx + b$.

SOLN: $5y - 4x = 17$ is equivalent to $y = \frac{4}{5}x + \frac{17}{5}$ whose slope is $4/5$. Thus any line parallel to this

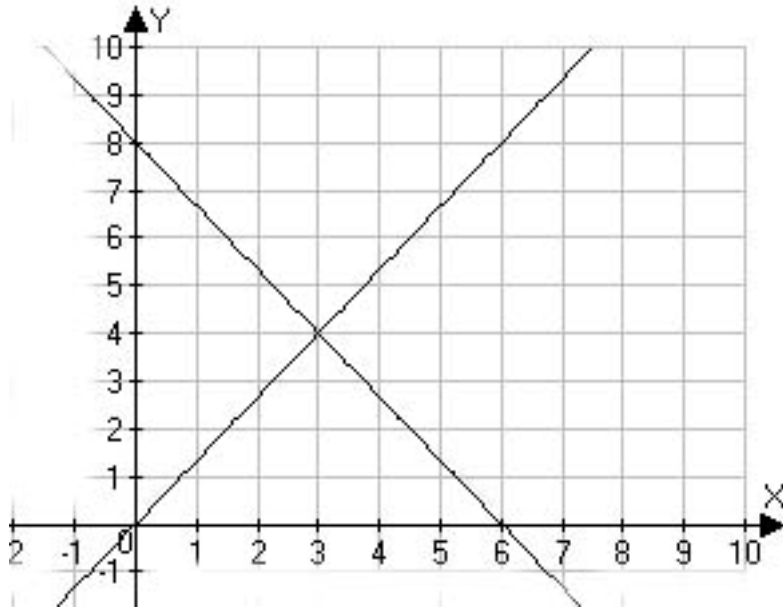
line will have a slope of $4/5$ and will be in the form $y = \frac{4}{5}x + b$.

6. Use the graphical method to solve the system of equations for x and y :

$$4x + 3y = 24$$

$$4x - 3y = 0$$

SOLN: Evidently, the lines intersect at $(3,4)$:



7. Write the equation of the horizontal line passing through $(7.2, -4.3)$. What are the coordinates of the point where this line intersects the line $2x + 3y = 60$?

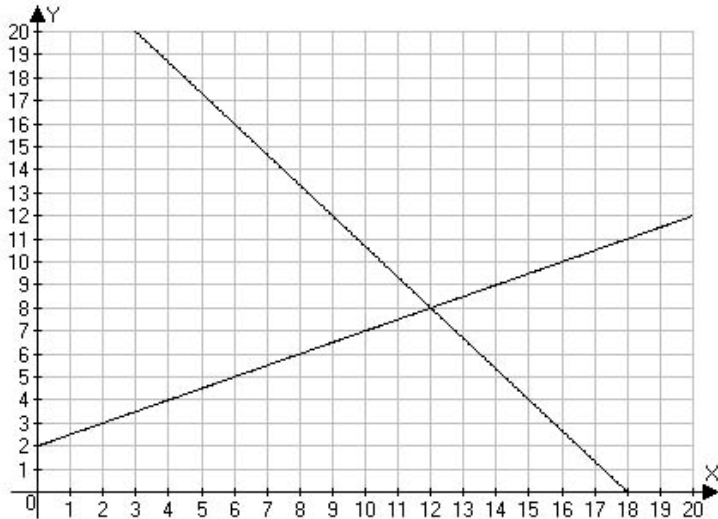
SOLN: The equation of the horizontal line passing through $(7.2, -4.3)$ is $y = -4.3$ and to find the x -coordinate of the point where this line intersects $2x + 3y = 60$, simply substitute $y = -4.3$ and solve for x : $2x + 3(-4.3) = 60$ so that $2x = 72.9$ or $x = 36.45$ and the point of intersection is $(36.45, -4.3)$

8. Solve the system of equations by graphing:

$$\frac{x}{3} + \frac{y}{4} = 6$$

$$2y = x + 4$$

SOLN: Evidently, the lines intersect at $(12,8)$.



9. Write a system of equations that is

a. dependent.

SOLN: $y = x$ and $2x - 2y = 0$ are equivalent equations, so the system is dependent.

b. inconsistent.

SOLN: $y = x$ and $y = x + 1$ are parallel, non-intersecting lines: an inconsistent system.

10. Solve the system of equations either by graphing, elimination or substitution. State whether the system is independent, dependent or inconsistent.

$$5x - y = 8$$

$$3x + 2y = 30$$

SOLN. Multiply both sides of the first equation by 2 to get a system with opposite coefficients for y .

Then equate the sum of the left hand sides with the sum of the right hand sides, thus eliminating y :

$$10x - 2y = 16$$

$$3x + 2y = 30$$

$$\hline 13x + 0 = 46$$

So $x = 46/13$. Substitute this back into the first equation above and solve for y

$$5\left(\frac{46}{13}\right) - y = 8 \Leftrightarrow y = \frac{230}{13} - 8 \Leftrightarrow y = \frac{126}{13} \quad \text{So the solution is } (x, y) = \left(\frac{46}{13}, \frac{126}{13}\right).$$