

Show your work for credit. Write all responses on separate paper.

Solve the word problems using the algebraic method:

Introduce a variable to represent an unknown.

Write related quantities in terms of the variable.

Write an equation using the variable.

Solve the equation.

State the solution and check that it's right.

1. A wooden board, 38 meters long, is cut into three pieces. The medium piece is 4 times as long as the shortest piece, and the longest piece is 2 meters longer than the medium piece. Find the length of the three pieces
2. Determine whether $a = 9$ satisfies the inequality $4 + 2(1 - a) > -4$
3. Joan earns a quarterly bonus of 3.9% on all sales exceeding \$15000. Determine what Joan's quarterly sales must be in order to earn a commission of at least \$770. Set $x =$ Joan's monthly sales in dollars.
4. Marc earns a yearly bonus of 1.6% of all sales he makes during the year in excess of \$75000. Determine what Marc's annual sales must be in order to earn a year-end bonus of at least \$1800.

5. Reduce to lowest terms:

a.
$$\frac{(10m^3)(-5mn)}{(25mn)(m^2)}$$

b.
$$\frac{-2x^2(8x^3)(5nx)}{6n^2x^3(5n^8x)}$$

6. Reduce the rational expression to a fraction in lowest terms: $\frac{10t^2 - 8t^2 - 3t^2}{5t^6 - 6t^6}$

7. Perform the indicated operations and simplify.

a.
$$\frac{a^2}{b} \left(\frac{ab}{c} \div \frac{c^2}{b^2} \right)$$

b.
$$\frac{x^2}{6} \div \left(\frac{2}{x} \div \frac{x}{3} \right)$$

8. If you need to equally divide $\frac{1}{8}$ of a cake among 7 people, how much of cake will each person receive?

9. Perform the indicated operation. Express your answer as a single fraction reduced to lowest terms.

a. $\frac{5}{x^3} - \frac{7}{3x}$

b. $3x^2 + \frac{1}{x} - \frac{7}{x^2}$

10. Solve the equation

a. $\frac{n}{3} + \frac{n+3}{6} = \frac{1}{2}$

b. $\frac{2x+3}{5} - \frac{x-3}{6} = \frac{68}{30}$

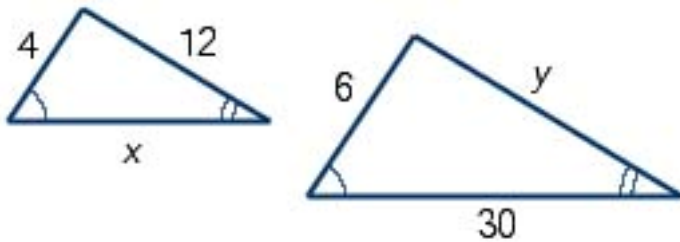
11. Solve the inequality

a. $\frac{n}{3} - \frac{n-3}{6} < \frac{1}{2}$

b. $\frac{2x-3}{5} - \frac{x+3}{6} \leq \frac{-2}{5}$

12. The ratio of length to width in a rectangle is $\frac{11}{5}$. If the perimeter is 160 meters, what are the length and the width?

13. The given triangles are similar. The equal angles are indicated by the same number of arcs within the angles. Find the lengths of the missing sides in each of the triangles.



14. A person drives from town A to town B at the rate of 30 mph, and then flies back at the rate of 190 mph. If the total traveling time is 22 hours, how far is it from town A to town B?

15. In a certain triangle, the medium side is 3 less than twice the shortest side, while the longest side is $2\frac{1}{2}$ times the shortest side. If the perimeter of the triangle is 2 more than 5 times the shortest side, find the lengths of the three sides of the triangle.

Math 54 – Elementary Algebra – Chapters 3 and 4 test Solutions

1. A wooden board, 38 meters long, is cut in three pieces. The medium piece is 4 times as long as the shortest piece, and the longest piece is 2 meters longer than the medium piece. Find the length of the 3 pieces

SOLN: Let x = the length of the shortest piece. Then the medium piece is $4x$ and the longest piece is $4x + 2$. The three pieces together make up the whole length, so $x + 4x + (4x + 2) = 38$. Combining like terms leads to the equivalent equation $9x + 2 = 38$. Subtracting 2 from each side yields a third equivalent equation, $9x = 36$ whose solution is obvious, $x = 4$. Thus the lengths are 4 meters, 16 meters and 18 meters, for a total of 38 meters.

2. Determine whether $a = 9$ satisfies the inequality $4 + 2(1 - a) > -4$

SOLN: Substituting, we have $4 + 2(1 - 9) = 4 + 2(-8) = 4 + (-16) = -12$ is not greater than -4 , so $a = 9$ does not satisfy the inequality.

3. Joan earns a quarterly bonus of 3.9% on all sales exceeding \$15000. Determine what Joan's quarterly sales must be in order to earn a commission of at least \$770. Set x = Joan's quarterly sales in dollars.

SOLN: The amount of sales exceeding 15000 is $x - 15000$. 3.9% of this is $0.039(x - 15000)$. If the commission is to be at least \$770, then $0.039(x - 15000) \geq 770 \Leftrightarrow 0.039x - 585 \geq 770$

So $0.039x \geq 1355 \Leftrightarrow 39x \geq 1355000 \Leftrightarrow x \geq \frac{1355000}{39} = \$34,743.59$

4. Marc earns a yearly bonus of 1.6% of all sales he makes during the year in excess of \$75000. Determine what Marc's annual sales must be in order to earn a year-end bonus of at least \$1800.

SOLN: Let x = Marc's annual sales. Then the amount in excess of \$75000 is $x - 75000$ so we seek x such that 1.6% of $(x - 75000)$ is at least 1800, that is,

$0.016(x - 75000) \geq 1800 \Leftrightarrow 0.016x - 1200 \geq 1800 \Leftrightarrow 0.016x \geq 3000 \Leftrightarrow 16x \geq 3000000$ thus $\Leftrightarrow x \geq 187500$

So Marc would need to make at least \$187,500 in sales.

5. Reduce to lowest terms:

a.
$$\frac{(10m^3)(-5mn)}{(25mn)(m^2)} = \frac{-2m(5 \cdot 5 \cdot m \cdot m \cdot m \cdot n)}{1(5 \cdot 5 \cdot m \cdot m \cdot m \cdot n)} = -2m$$

b.
$$\frac{-2x^2(8x^3)(5nx)}{6n^2x^3(5n^8x)} = \frac{-2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot n}{2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n} = \frac{-8x^2}{3n^9}$$

6. Reduce the rational expression to a fraction in lowest terms:

$$\frac{10t^2 - 8t^2 - 3t^2}{5t^6 - 6t^6} = \frac{(10 - 8 - 3)t^2}{(5 - 6)t^6} = \frac{-t^2}{-t^6} = \frac{1 \cdot t \cdot t}{t \cdot t \cdot t \cdot t \cdot t \cdot t} = \frac{1}{t^4}$$

7. Perform the indicated operations and simplify.

a.
$$\frac{a^2 \left(\frac{ab}{c} \div \frac{c^2}{b^2} \right)}{b \left(\frac{ab}{c} \cdot \frac{b^2}{c^2} \right)} = \frac{a^2 \left(\frac{ab}{c} \cdot \frac{b^2}{c^2} \right)}{b \left(\frac{ab}{c} \cdot \frac{b^2}{c^2} \right)} = \frac{a \cdot a \cdot a \cdot b \cdot b \cdot b}{b \cdot c \cdot c \cdot c} = \frac{a^3 b^2}{c^3}$$

b.
$$\frac{x^2}{6} \div \left(\frac{2}{x} \div \frac{x}{3} \right) = \frac{x^2}{6} \div \left(\frac{2}{x} \cdot \frac{3}{x} \right) = \frac{x^2}{6} \div \left(\frac{6}{x^2} \right) = \frac{x^2}{6} \cdot \left(\frac{x^2}{6} \right) = \frac{x^4}{36}$$

8. If you need to equally divide $\frac{1}{8}$ of a cake among 7 people, how much of cake will each person receive?

SOLN: Each person will get $\frac{1}{7}$ of $\frac{1}{8}$ of the cake, or $\frac{1}{56}$ of the cake. Those are small portions!

9. Perform the indicated operation. Express your answer as a single fraction reduced to lowest terms.

a.
$$\frac{5}{x^3} - \frac{7}{3x} = \left(\frac{3}{3}\right) \cdot \frac{5}{x^3} - \frac{7}{3x} \cdot \left(\frac{x^2}{x^2}\right) = \frac{15 - 7x^2}{3x^3}$$

b.
$$3x^2 + \frac{1}{x} - \frac{7}{x^2} = \frac{3x^2}{1} \cdot \left(\frac{x^2}{x^2}\right) + \frac{1}{x} \cdot \left(\frac{x}{x}\right) - \frac{7}{x^2} = \frac{3x^4 + x - 7}{x^2}$$

10. Solve the equation

a. $\frac{n}{3} + \frac{n+3}{6} = \frac{1}{2}$ SOLN: Multiply both sides of the equation by the least common multiple of 3, 6 and 2:

$$6\left(\frac{n}{3} + \frac{n+3}{6}\right) = 6 \cdot \frac{1}{2} \Leftrightarrow 2n + (n+3) = 3 \Leftrightarrow 3n + 3 = 3 \Leftrightarrow 3n = 0 \Leftrightarrow \boxed{n=0}$$

b.
$$\frac{2x+3}{5} - \frac{x-3}{6} = \frac{68}{30}$$

SOLN:
$$30\left(\frac{2x+3}{5} - \frac{x-3}{6}\right) = 30 \cdot \frac{68}{30} \Leftrightarrow 6(2x+3) - 5(x-3) = 68 \Leftrightarrow 12x + 18 - 5x + 15 = 68$$

$$\Leftrightarrow 7x + 33 = 68 \Leftrightarrow 7x = 35 \Leftrightarrow \boxed{x=5}$$

11. Solve the inequality

a.
$$\frac{n}{3} - \frac{n-3}{6} < \frac{1}{2} \Leftrightarrow 6\left(\frac{n}{3} - \frac{n-3}{6}\right) < 6 \cdot \frac{1}{2} \Leftrightarrow 2n - (n-3) < 3 \Leftrightarrow n + 3 < 3 \Leftrightarrow \boxed{n < 0}$$

b.
$$\frac{2x-3}{5} - \frac{x+3}{6} \leq \frac{-2}{5}$$

$$\Leftrightarrow 30\left(\frac{2x-3}{5} - \frac{x+3}{6}\right) \leq 30\left(\frac{-2}{5}\right) \Leftrightarrow 6(2x-3) - 5(x+3) \leq -12 \Leftrightarrow 7x - 33 \leq -12 \Leftrightarrow 7x \leq 11 \Leftrightarrow \boxed{x \leq \frac{11}{7}}$$

12. The ratio of length to width in a rectangle is $\frac{11}{5}$. If the perimeter is 160 meters,

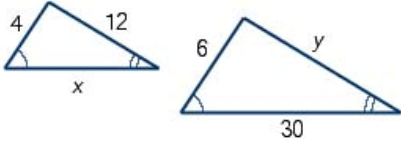
what are the length and the width?

Let L = the length of the rectangle. Then the width is $80 - L$ and so

$$\frac{L}{80-L} = \frac{11}{5} \Leftrightarrow 5L = 11(80-L) \Leftrightarrow 5L = 880 - 11L \Leftrightarrow 16L = 880 \Leftrightarrow L = 55 \text{ meters and}$$

so the width is 25 meters.

13. The given triangles are similar. The equal angles are indicated by the same number of arcs within the angles. Find the lengths of the missing sides in each of the triangles.



SOLN: $\frac{x}{4} = \frac{30}{6} \Leftrightarrow \boxed{x = 20}$ and $\frac{y}{6} = \frac{12}{4} \Leftrightarrow \boxed{y = 18}$

14. A person drives from town A to town B at the rate of 30 mph, and then flies back at the rate of 190 mph. If the total traveling time is 22 hours, how far is it from town A to town B ?

SOLN: Let x = the distance from A to B . Then the time going from A to B is $x/30$ and the time going from B to A is $x/190$ so that the total time is

$$\frac{x}{30} + \frac{x}{190} = 22 \Leftrightarrow 570 \left(\frac{x}{30} + \frac{x}{190} \right) = 22 \cdot 570 \Leftrightarrow 19x + 3x = 22 \cdot 570 \Leftrightarrow 22x = 22 \cdot 570 \Leftrightarrow \boxed{x = 570} \text{ miles.}$$

15. In a certain triangle, the medium side is 3 less than twice the shortest side, while the longest side is $2\frac{1}{2}$ times the shortest side. If the perimeter of the triangle is 2 more than 5 times the shortest side,

find the lengths of the three sides of the triangle.

SOLN: Let x = the length of the shortest side. Then the length of the medium side is $2x - 3$ and the longest side has length $2.5x$ so that the perimeter is $x + (2x - 3) + 2.5x = 5x + 2$. This is equivalent to $5.5x - 3 = 5x + 2$ so that $0.5x = 5$ and $x = 10$, $2x - 3 = 17$ and $2.5x = 25$ are the lengths of the three sides.