Math 54–Beginning Algebra **Name (Print):** _____

Spring 2015 : 3/5/15
Exam 2 Solutions: Chapters 5 & 6

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1. (15 points) Simplify each expression. Answers for (a) and (b) should contain positive exponents only. In (c) write the answer in scientific notation.

   (a) $-4^3 + (-2)^6 = -64 + 64\boxed{= 0}$

   (b) $\dfrac{(4x^2)^{-1}(x^{-2})^5}{(2x^3)^{-4}} = \dfrac{x^{-10}}{(4x^2)(2^{-4}x^{-12})} = \dfrac{2^4x^2}{4x^2} = \dfrac{16}{4}\boxed{= 4}$

   (c) $\dfrac{(6.0 \times 10^4)3.0 \times 10^{-2}}{3.6 \times 10^{-6}} = \dfrac{6.0(3.0)}{3.6} \times \dfrac{10^4 10^{-2}}{10^{-6}}\boxed{= 5 \times 10^8}$

2. (16 points) Given two polynomials, $A = x^2 + 2x + 4$ and $B = x^2 - 2x + 4$.

   (a) Substitute and simplify $(A - B)(A + B)$
   ANS: $(A - B)(A + B) = (x^2 + 2x + 4 - (x^2 - 2x + 4))(x^2 + 2x + 4 + x^2 - 2x + 4)$
   $= 4x(2x^2 + 8)\boxed{= 8x(x^2 + 4) = 8x^3 + 32x}$

   (b) Substitute and simplify $A \cdot B$
   ANS: $A \cdot B = (x^2 + 2x + 4)(x^2 - 2x + 4) = x^2(x^2 - 2x + 4) + 2x(x^2 - 2x + 4) + 4(x^2 - 2x + 4) =$
   $= x^4 - 2x^3 + 4x^2 + 2x^3 - 4x^2 + 8x + 4x^2 - 8x + 16\boxed{= x^4 + 4x^2 + 16}$

   (c) Evaluate both A and B when $x = 4$
   $A = 4^2 + 2(4) + 4 = 28$ and $B = 4^2 - 2(4) + 4 = 12$

   (d) Evaluate both $(A - B)(A + B)$ and $A \cdot B$ when $x = 4$. Check your answers in parts (a) and (b).
   ANS: $(A - B)(A + B) = (28 - 12)(28 + 12) = 16 \cdot 40 = 640$.
   Also, $8 \cdot 4^3 + 32 \cdot 4 = 8 \cdot 64 + 128 = 512 + 128 = 640$.
   $A \cdot B = 28 \cdot 12 = 316$.
   Also $4^4 + 4 \cdot 4^2 + 16 = 256 + 64 + 16 = 316$.

3. (14 points) Divide using long division. Relate the dividend, divisor, quotient and remainder in an equation.

   (a) $\dfrac{12x^2 - 23x + 10}{4x - 5}$

   $$\begin{array}{r} 3x - 2 \\ 4x - 5\overline{)\ 12x^2 - 23x + 10} \\ \underline{-\,12x^2 + 15x\ \ \ \ } \\ -\,8x + 10 \\ \underline{\ \ 8x - 10} \\ 0 \end{array}$$

   $\dfrac{12x^2 - 23x + 10}{4x - 5} = 3x - 2$

   (b) $\dfrac{24x^3 - 82x^2 + 89x - 30}{2x - 3}$

   $$\begin{array}{r} 12x^2 - 23x + 10 \\ 2x - 3\overline{)\ 24x^3 - 82x^2 + 89x - 30} \\ \underline{-\,24x^3 + 36x^2\ \ \ \ \ \ } \\ -\,46x^2 + 89x \\ \underline{\ \ 46x^2 - 69x} \\ 20x - 30 \\ \underline{-\,20x + 30} \\ 0 \end{array}$$

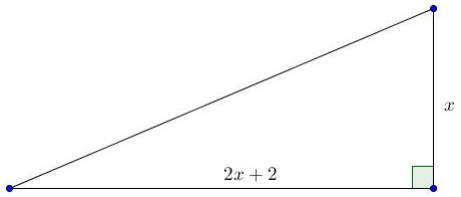
   $\dfrac{24x^3 - 82x^2 + 89x - 30}{2x - 3} = 12x^2 - 23x + 10$

4. (15 points) Factor completely.
   (a) $2t^2 - 24t + 22 = 2(t^2 - 12t + 11) = 2(t^2 - t - 11t + 11) = 2(t(t-1) - 11(t-1))\boxed{= 2(t-11)(t-1)}$
   (b) $x^2 - 9y^2 = x^2 - (3y)^2\boxed{= (x-3y)(x+3y)}$
   (c) $3B^3 - 81 = 3(B^3 - 27) = 3(B^3 - 3^3)\boxed{= 3(B-3)(B^2+3B+9)}$

5. (15 points) Use the zero product principle to find *all* solutions for each equation.
   (a) $x^2 - 5x - 84 = 0$
   $(x-12)(x+7) = 0$
   $\boxed{x = 12 \text{ or } x = -7}$
   (b) $6t^2 = 23t - 20$
   $6t^2 - 23t + 20 = 0$
   $6t^2 - 15t - 8t + 20 = 0$
   $3t(2t-5) - 4(2t-5) = 0$
   $(3t-4)(2t-5) = 0$
   $\boxed{t = \frac{4}{3} \text{ or } t = \frac{5}{2}}$
   (c) $8A^3 = 98A$
   $2A(4A^2 - 49) = 0$
   $2A(2A-7)(2A+7) = 0$
   $\boxed{A = 0 \text{ or } A = \pm\frac{7}{2}}$

6. (10 points) One number is 3 more than twice another number. The product of the numbers is 5 more than their sum. Use the algebraic method to find all such numbers.
   ANS: Let $x =$ the smaller number. Then the larger number is $2x + 3$. The product of the numbers is $x(2x+3)$ and 5 more than their sum is $x + (2x+3) + 5$. Thus we have the equation
   $x(2x+3) = x + (2x+3) + 5 \Leftrightarrow 2x^2 + 3x = 3x + 8 \Leftrightarrow 2x^2 = 8 \Leftrightarrow x^2 - 4 = 0 \Leftrightarrow (x-2)(x+2) = 0.$
   So the numbers could be 2 and 7 or -2 and -1.

7. (15 points) The longer leg of a right triangle is 2 more than twice the shorter leg. The hypotenuse is 2 less than three times the shorter leg.
   (a) If we let $x =$ the length of the shorter leg, then the longer leg is $2x + 2$, as shown in the diagram. Express the hypotenuse in terms of the shorter leg.
   ANS: The hypotenuse is $3x - 2$

   (b) Use the Pythagorean theorem to write an equation involving x.
   $x^2 + (2x+2)^2 = (3x-2)^2$

   (c) Solve the equation.
   $x^2 + 4x^2 + 8x + 4 = 9x^2 - 12x + 4 \Leftrightarrow 9x^2 - 12x + 4 = 5x^2 + 8x + 4$
   $\Leftrightarrow 4x^2 - 20x = 0 \Leftrightarrow 4x(x-5) = 0$ So $x = 0$ or $\boxed{x = 5}$

   (d) What are the lengths of the sides of the right triangle?
   5, 12 and 13.