

**Math 54 Sprint 2015 Exam 1 Solutions: Chapters 1 & 2 2/12/15**

1. (12 points) Simplify.

(a)  $9(3N + 5) - (8N - 7) = 27N + 45 - 8N + 7 = \boxed{19N + 52}$

(b)  $10\left(\frac{1}{2}x + \frac{1}{5}y\right) + 9\left(x + \frac{2}{3}y\right) = 5x + 2y + 9x + 6y = \boxed{14x + 8y}$

2. (12 points) Solve each equation.

(a)  $-5(m-3) + 2(3m+1) = 15 - 8m \Leftrightarrow -5m + 15 + 6m + 2 = 15 - 8m \Leftrightarrow m + 17 = 15 - 8m \Leftrightarrow 9m = -2 \Leftrightarrow \boxed{m = -\frac{2}{9}}$

(b)  $3x + \frac{1}{3} = 5x - \frac{2}{3} \Leftrightarrow -2x = -1 \Leftrightarrow \boxed{x = \frac{1}{2}}$

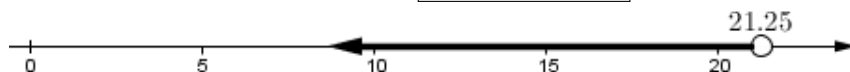
3. (12 points) Solve each equation for  $y$ .

(a)  $\frac{2}{3}(y+3) = (x-2) \Leftrightarrow \frac{2}{3}y + 2 = x - 2 \Leftrightarrow \frac{2}{3}y = x - 4 \Leftrightarrow \boxed{y = \frac{3}{2}x - 6}$

(b)  $\frac{2y+1}{2x} = -\frac{1}{2} \Leftrightarrow 2y+1 = -x \Leftrightarrow 2y = -x-1 \Leftrightarrow \boxed{y = -\frac{x+1}{2}}$

4. (12 points) Solve each inequality and graph the solution on the number line.

(a)  $5 - \frac{4}{5}y > -12 \Leftrightarrow -\frac{4}{5}y > -17 \Leftrightarrow \boxed{y < \frac{85}{4} = 21.25}$



(b)  $0.6(T-3) \leq 0.3 \Leftrightarrow 0.6T - 1.8 \leq 0.3 \Leftrightarrow 6T - 18 \leq 3 \Leftrightarrow 6T \leq 21 \Leftrightarrow T \leq \frac{7}{2} \Leftrightarrow \boxed{T \leq 3.5}$



5. (13 points) One side of a triangle is half the longest side. The third side is 10 meters less than the longest side. The perimeter is 45 meters. Follow the steps below to find all three sides.

(a) Identify an unknown, be specific. That is, let  $x$  = one of the sides, and be specific as to which side you mean.

SOLN: Let  $x$  = the longest side

(b) Right the lengths of the other two sides in terms of your  $x$ .

SOLN: Then the other two sides have lengths  $x/2$  and  $x - 10$

(c) Set up an equation involving  $x$ .

$$x + \frac{x}{2} + (x - 10) = 45$$

(d) Solve your equation.

SOLN: Clearing the fraction,  $2x + x + 2x - 20 = 90 \Leftrightarrow 5x - 20 = 90 \Leftrightarrow 5x = 110 \Leftrightarrow x = 22$

(e) State the answer to the problem. SOLN: The sides have lengths 22, 11 and 12.

(f) Check your answer.

$$\text{SOLN: } 22 + 11 + 12 = 45$$

6. (13 points) Travis has a savings account that his parents opened for him. It pays 2% annual interest. His uncle also opened an account for him, but it pays 3% annual interest. If there is \$800 more in the account that pays 3%, and the total interest from both accounts is \$74, follow the steps below to determine how much money is in each of the accounts.

(a) Let  $x$  = the amount invested at 2%. (identify your unknown as the amount invested in one account or the other)

(b) Then  $x + 800$  = the amount invested at 3%. (write the amount invested in the other account in terms of your unknown.)

(c) Write the interest earned on each account in terms of the unknown.

The interest after one year on  $x$  dollars invested at 2% is  $0.02 \cdot x$  and the interest after one year on  $x + 800$  invested at 3% is  $0.03(x + 800)$

(d) Write an equation involving the unknown.

$$\text{The total interest accrued after one year is } 0.02 \cdot x + 0.03(x + 800) = 74$$

(e) Solve the equation and check your answer.

Expanding the left hand side, combining like terms and then using the properties of equality to solve for  $x$  we have

$$0.02x = 0.03x + 24 = 74 \Leftrightarrow 0.05x + 24 = 74 \Leftrightarrow 0.05x = 50 \Leftrightarrow x = 1000. \text{ So the amount invested at 2\% is \$1000 and the amount invested at 3\% is \$1800. This checks out since } 0.02(1000) + 0.03(1800) = 20 + 54 = 74.$$

7. (14 points) The length and width of a rectangle are consecutive odd integers. The perimeter is 128 meters. Find the length and width.

SOLN. Let  $x$  = the width of the rectangle. Then  $x + 2$  = the length and the perimeter is  $2x + 2(x + 2) = 128 \Leftrightarrow 2x + 2x + 4 = 128 \Leftrightarrow 4x + 4 = 128 \Leftrightarrow 4x = 124 \Leftrightarrow x = 31$ . So the width is 31 and the length is 33. This checks since  $62 + 66 = 128$ .

8. (12 points) Solve the following compound inequalities. Graph the solution set in each case.

(a)  $-7 \leq 2x - 3 \leq 7 \Leftrightarrow -4 \leq 2x \leq 10 \Leftrightarrow \boxed{-2 \leq x \leq 5}$ .



(b)  $11x - 8 > 3$  or  $12x + 7 < -5 \Leftrightarrow 12x < -12$  or  $11x > 11 \Leftrightarrow x < -1$  or  $x > 1$

