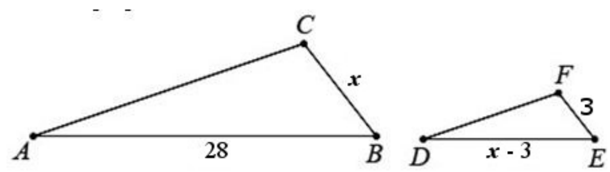
Math 54–Beginning Algebra, Fall 2014 : 11/25/14 Name (Print): ____________________

Exam 4: Chapters 7 & 8

Instructions: Show your work for credit. Write all responses on separate paper.

1. Identify each algebraic statement as an expression or an equation. If the statement is an expression, perform the indicated operations and simplify. If the statement is an equation solve it.
   (a) $\frac{4}{2x-5}+\frac{1}{x}=1 \Leftrightarrow 4x+2x+5=x(2x+5) \Leftrightarrow 6x-5=2x^2-5x \Leftrightarrow 2x^2-11x+5=0 \Leftrightarrow$ $(2x-1)(x-5)=0 \Leftrightarrow x=\frac{1}{2}$ or $x=5$
   This is an equation.
   (b) $\left(\frac{2}{x}+\frac{1}{2x-2}\right)\div\frac{5x-4}{x}=\frac{2(2x-2)+x}{x(2x-2)}\cdot\frac{x}{5x-4}=\frac{5x-4}{x(2x-2)}\cdot\frac{x}{5x-4}=\frac{1}{2x-2}$
   This is an expression.

2. In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. That means that their corresponding sides are proportional.



   (a) Use the fact that the ratio of $\overline{BC}$ to $\overline{AB}$ is equal to the ratio of $\overline{EF}$ to $\overline{DE}$ to write an equation involving x.
   SOLN: $\frac{x}{28}=\frac{3}{x-3}$
   (b) Solve the equation to find length of $\overline{BC}$.
   SOLN: $\frac{x}{28}=\frac{3}{x-3} \Leftrightarrow x(x-3)=84 \Leftrightarrow x^2-3x-84=0$
   Here is a listing of the possible ways of factoring -84:

Factors of -84	$-84\cdot 1$	$-42\cdot 2$	$-28\cdot 3$	$-21\cdot 4$	$-14\cdot 6$	$-12\cdot 7$
sum of factors	-83	-40	-25	-17	-8	-5

   Since there are not two integers whose product is -84 which add up to -3, we conclude that the quadratic $x^2-3x-84$ is prime. However, we can always solve a quadratic using the method of completing the square: $x^2-3x=84 \Leftrightarrow x^2-3x+\frac{9}{4}=84+\frac{9}{4} \Leftrightarrow \left(x-\frac{3}{2}\right)^2=\frac{345}{4}$
   $\Leftrightarrow x-\frac{3}{2}=\pm\frac{\sqrt{345}}{2} \Leftrightarrow x=\frac{3\pm\sqrt{345}}{2}$

3. Recall that $\sqrt[3]{x}$ is defined as the real number whose cube is x.
   (a) Simplify the expression$(\sqrt[3]{x}-\sqrt[3]{27})^2$ for $x=8$.
   SOLN: $(\sqrt[3]{8}-\sqrt[3]{27})^2=(2-3)^2=(-1)^2=1$
   (b) Find all values of x so that $\sqrt[3]{x^2+1}-\sqrt[3]{64}=0$ and simplify these.
   SOLN: $\sqrt[3]{x^2+1}-\sqrt[3]{64}=0 \Leftrightarrow \sqrt[3]{x^2+1}=\sqrt[3]{64} \Leftrightarrow x^2+1=64 \Leftrightarrow x^2=63 \Leftrightarrow x=\pm 3\sqrt{7}$

4. It takes Noel 1/2 an hour longer to walk 5 miles up a hill than to walk 4 miles back down the hill. If his speed walking downhill is 3 mph faster than his speed walking uphill, what is his speed walking uphill? Fill in a chart like this one to answer help set up an equation you solve to answer the question.

	Distance	Rate	Time
uphill	5	x	$\frac{5}{x}$
downill	4	$x+3$	$\frac{4}{x+3}$

so that

$$\boxed{\begin{gathered}\frac{5}{x}=\frac{4}{x+3}+\frac{1}{2} \Leftrightarrow 10(x+3)=8x+x(x+3)\\ \Leftrightarrow x^2+x-30=0 \Leftrightarrow (x+6)(x-5)=0\\ \text{Noel walks uphill at 5 mph}\end{gathered}}$$

5. (15 points) Simplify each expression:

   (a) $\dfrac{2}{\sqrt{2}} + \dfrac{3}{\sqrt{8}} = \sqrt{2} + \dfrac{3}{2\sqrt{2}} = \sqrt{2} + \dfrac{3\sqrt{2}}{4} = \dfrac{4\sqrt{2}+3\sqrt{2}}{4} = \dfrac{7\sqrt{2}}{4}$

   (b) $(1-\sqrt{10})(1+\sqrt{10}) = 1 - 10 = -9$

   (c) $\dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}+\sqrt{5}} = \dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}+\sqrt{5}}\dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}} = \dfrac{x-2\sqrt{5x}-5}{x-5}$

6. (16 points) Solve the following equations

   (a) $\sqrt{x^2+6x} = 4 \Rightarrow x^2+6x = 16 \Leftrightarrow x^2+6x-16 = 0 \Leftrightarrow (x+8)(x-2) = 0 \Leftrightarrow x = -8$ or $x = 2$ Both solutions work.

   (b) $\sqrt{2x+6} = x-1 \Rightarrow 2x+6 = (x-1)^2 \Leftrightarrow 2x+6 = x^2-2x+1 \Leftrightarrow x^2-4x-5 = 0 \Leftrightarrow (x-5)(x+1) = 0 \Leftrightarrow x = 5$ or $x = -1$ Only $x = 5$ works.

7. (13 points) The legs of a right triangle have lengths $\sqrt{2}-1$ and $\sqrt{2}+1$ Compute and simplify the length of the hypotenuse using Pythagoras' theorem.
   $c^2 = (\sqrt{2}-1)^2 + (\sqrt{2}+1)^2 = 2 - 2\sqrt{2} + 1 + 2 + \sqrt{2} + 1 = 6 \Rightarrow c = \sqrt{6}$