Math 54–Beginning Algebra, Fall 2014 : 10/29/14 Exam 3 Solutions: Chapters 3 & 4

1. (14 points) Construct a table of values for

   (a) the equation $3x - 5y = 15$. Include points where $x = 0, x = -5, y = 0, y = 3$
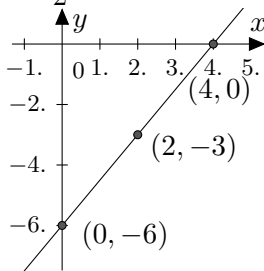
   SOLN:

x	0	−5	5	10
y	−3	−6	0	3

   (b) the equation $y = -\dfrac{2}{5}x + 3$. Include points where $x = 0, x = 3, y = 0, y = 3$

   SOLN:

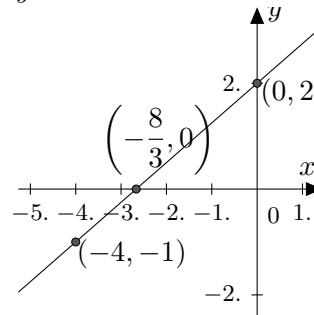 | x | 0 | 3 | $\dfrac{15}{2}$ |
   |---|---|---|---|
   | y | 3 | $\dfrac{9}{5}$ | 0 |

2. (14 points) Construct a careful graph for each of the following equations. Include the intercepts and a third corroborating point.

   (a) $y = \dfrac{3}{2}x - 6$

   (b) $4y - 3x = 8$

3. (14 points) Find the slope-intercept form for the equation that fits the tabulated solutions.

   (a)

x	0	2	6
y	−2	1	7

   SOLN: The slope is $\dfrac{1 - (-2)}{2 - 0} = \dfrac{3}{2}$
   and the y-intercept is -2,
   so we write $\boxed{y = \dfrac{3}{2}x - 2}$

   (b)

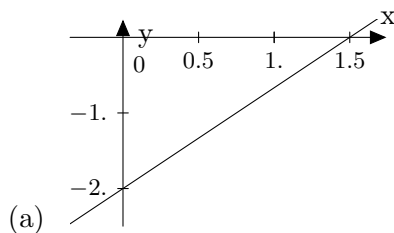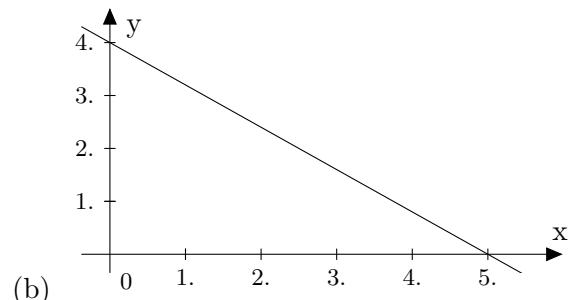 | x | 1 | 5/2 | 4 |
   |---|---|---|---|
   | y | −1 | 0 | 1 |

   SOLN: The slope is $\dfrac{1 - (-1)}{4 - 1} = \dfrac{2}{3}$
   Using the point-slope formula, we have
   $$y - (-1) = \dfrac{2}{3}(x - 1) \Leftrightarrow \boxed{y = \dfrac{2}{3}x - \dfrac{5}{3}}$$

4. (14 points) Find an equation for the line perpendicular to the line whose graph is shown and having the same y−intercept.
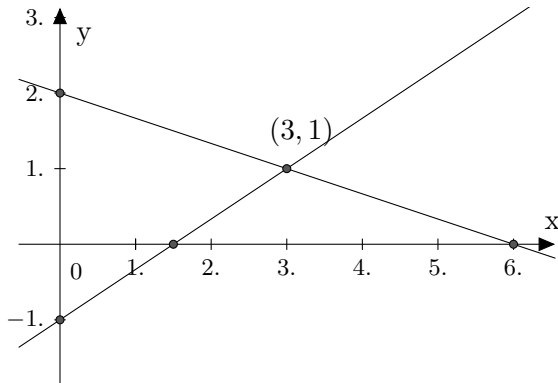
   (a)

   $m = \dfrac{2}{1.5} = \dfrac{4}{3} \Rightarrow m_\perp = -\dfrac{3}{4}$ and the

   y-intercept is -2 so $\boxed{y = -\dfrac{3}{4}x - 2}$
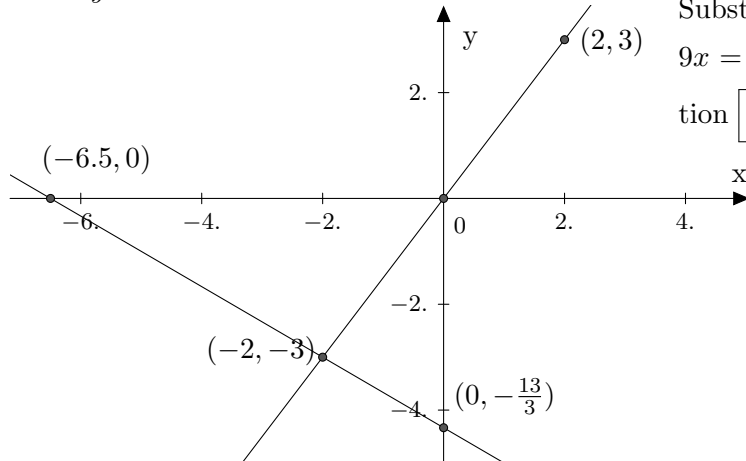
   (b)

   $m = -\dfrac{4}{5} \Rightarrow m_\perp = \dfrac{5}{4}$ and the y-intercept is 4

   So $\boxed{y = \dfrac{5}{4}x + 4}$

5. (14 points) Solve each system by graphing.

(a) $x + 3y = 6$
$2x - 3y = 3$

(3, 1)

6. (14 points) Solve each system using either elimination or subsitution: your choice.

(a) $4x - 2y = 6$
$5x - 3y = 2$
Multiplying the first equation by -3 and the second by 2, we have
$-12x + 6y = -18$
$10x - 6y = 4$
Whence equating the sums of left and right sides we have $-2x = -14 \Leftrightarrow x = 7$. Substitute this to find $y = 11$. So $\boxed{(x, y) = (7, 11)}$ is the solution. It checks.

(b) $3x - 2y = 0$
$2x + 3y = -13$

$(-6.5, 0)$
$(-2, -3)$
$(2, 3)$
$(0, -\frac{13}{3})$

(b) $2y - 3x = 8$
$y = 6x + 1$
Substituting, we have $2(6x + 1) - 3x = 8 \Leftrightarrow$
$9x = 6 \Leftrightarrow x = \dfrac{2}{3}$ whence $y = 5$ and the solution $\boxed{(x, y) = (\frac{2}{3}, 5)}$ checks nicely.

7. (16 points) Sammy has money in two accounts: some invested at 6% annual interest and the rest invested at 5%. The total investment is $2800 and the total interest payed from the two accounts after a year is $156. Use the algebraic method to set up two equations in two unknowns and solve the system to determine how much was invested in each account.
SOLN: Let $x = $ the amount invested at 6% and $y = $ the amount invested at 5%.
The interest paid on x dollars invested at 5% is $0.05x$ while
the interest paid on y dollars invested at 6% is $0.06y$.
Thus we get the system of equations

$$x + y = 2800$$
$$0.05x + 0.06y = 156$$

Using the mutliplication property of equality, we get the equivalent system

$$-5x - 5y = -14000$$
$$5x + 6y = 15600$$

Equating the sums of left and right sides yields $y = 1600$. To make up the total investment then, $x = 1200$. Thus Sammy invested $1200 at 5% annual interest and $1600 at 6%, yielding a total of $0.05 \cdot 1200 + 0.06 \cdot 1600 = 60 + 96 = 156$.