Fall 2014 : 10/9/14
Exam 2: Chapters 5 & 6 - Solutions

---

1. (15 points) Simplify each expression. Answers for a) and (b) should contain positive exponents only. In (c) write the answer in scientific notation.
   (a) $-4^3 - 7^0 + 7^1 = -64 - 1 + 7 = -58$
   (b) $\dfrac{(x^3)^{-2}(x^{-1})^5}{(x^{-3})^{-4}} = \dfrac{x^{-6}x^{-5}}{x^{12}} = \dfrac{x^{-11}}{x^{12}} = \dfrac{1}{x^{23}}$
   (c) $\dfrac{1.5 \times 10^{-2}}{7.5 \times 10^{-8}} = \dfrac{1.5}{7.5} \times \dfrac{10^{-2}}{10^{-8}} = 0.2 \times 10^6 = 2 \times 10^5$

2. (16 points) Given two polynomials, $A = 7x^2 + 3x - 6$ and $B = 5x^2 - 4x + 7$.
   (a) Substitute and simplify $A - B = 7x^2 + 3x - 6 - (5x^2 - 4x + 7) = 2x^2 + 7x - 13$
   (b) Substitute and simplify $A \cdot B = (7x^2 + 3x - 6)(5x^2 - 4x + 7) = 35x^4 - 28x^3 + 15x^3 + 49x^2 - 12x^2 - 30x^2 + 24x + 21x - 42 = 35x^4 - 13x^3 + 7x^2 + 45x - 42$
   To be sure, try the grid method:

$\times$	$7x^2$	$3x$	-6
$5x^2$	$35x^4$	$15x^3$	$-30x^2$
$-4x$	$-28x^3$	$-12x^2$	$24x$
7	$49x^2$	$21x$	-42

   (c) Evaluate both A and B when $x = 2$.
   SOLN: $A = 7 \cdot 2^2 + 3 \cdot 2 - 6 = 28 + 6 - 6 = \boxed{28}$, $B = 5 \cdot 2^2 - 4 \cdot 2 + 7 = 20 - 8 + 7 = \boxed{19}$
   (d) Evaluate both $A - B$ and $A \cdot B$ when $x = 2$. This should be a check for parts (a) and (b).
   SOLN: When $x = 2$, $\boxed{A - B = 2 \cdot 2^2 + 7 \cdot 2 - 13 = 8 + 14 - 13 = 9}$, consistent with $28 - 19 = 9$.
   Also, when $x = 2$, $\boxed{A \cdot B = 35 \cdot 2^4 - 13 \cdot 2^3 + 7 \cdot 2^2 + 45 \cdot 2 - 42}$
   $\boxed{= 35 \cdot 16 - 13 \cdot 8 + 7 \cdot 4 + 45 \cdot 2 - 42 = 560 - 104 + 28 + 90 - 42 = 532}$. Also, $28 \cdot 19 = 532$.

3. (14 points) Do long division. Relate the dividend, divisor, quotient and remainder in an equation.

   (a) $\dfrac{3x^2 - 10x + 7}{3x - 2}$

$$
\begin{array}{r|l}
 & x - \frac{8}{3} \\
\hline
3x - 2 & 3x^2 - 10x + 7 \\
 & 3x^2 - 2x \\
\hline
 & -8x + 7 \\
 & -8x + \frac{16}{3} \\
\hline
 & \frac{5}{3}
\end{array}
$$

   Thus $3x^2 - 10x + 7 = (3x - 2)\left(x - \dfrac{8}{3}\right) + \dfrac{5}{3}$

   (b) $\dfrac{2x^3 - 7x^2 + 6x + 10}{x + 1}$

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 15 \\
\hline
x + 1 & 2x^3 - 7x^2 + 6x + 10 \\
 & 2x^3 + 2x^2 \\
\hline
 & -9x^2 + 6x \\
 & -9x^2 - 9x \\
\hline
 & 15x + 10 \\
 & 15x + 15 \\
\hline
 & -5
\end{array}
$$

   $2x^3 - 7x^2 + 6x + 10 = (x + 1)(2x^2 - 9x + 15) - 5$

4. (15 points) Factor completely.

   (a) $3y^2 + 3y - 18 = 3(y^2 + y - 6) = 3(y^2 + (3y - 2y) - 6) = 3\left[y(y+3) - 2(y+3)\right] = \boxed{3(y+3)(y-2)}$

   (b) $4x^2 - 9 = (2x)^2 - 3^2 = (2x)^2 + 6x - 6x + 3^2 = 2x(2x+3) - 3(2x+3) = \boxed{(2x+3)(2x-3)}$

   (c) $2 - 54A^3 = 2(1 - 27A^3) = 2(1^3 - (3A)^3) = \boxed{2(1-3A)(1+3A+9A^2)}$

5. (15 points) Use the zero product principle to find *all* solutions for each equation.

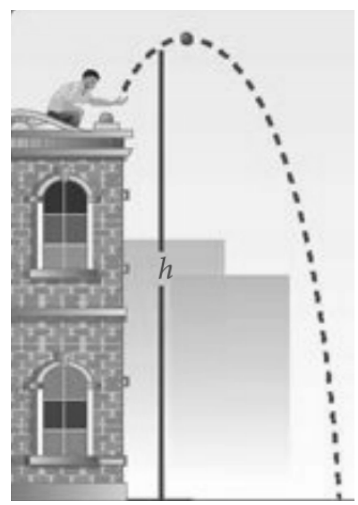
   (a) $x^2 + 2x - 63 = 0 \Leftrightarrow x^2 + 9x - 7x - 63 = 0 \Leftrightarrow x(x+9) - 7(x+9) = 0 \Leftrightarrow (x+9)(x-7) = 0$
   $\boxed{\Leftrightarrow x = 7 \text{ or } x = -9.}$

   (b) $2t^2 = 7t + 15 \Leftrightarrow 2t^2 - 7t - 15 = 0 \Leftrightarrow 2t^2 - 10t + 3t - 15 = 0 \Leftrightarrow 2t(t-5) + 3(t-5)$
   $\Leftrightarrow (t-5)(2t+3) = 0 \boxed{\Leftrightarrow t = 5 \text{ or } t = -\frac{3}{2}}$

   (c) $4y^3 = 25y \Leftrightarrow 4y^3 - 25y = 0 \Leftrightarrow y(4y^2 - 25) = 0 \Leftrightarrow y(2y-5)(2y+5) = 0 \Leftrightarrow \boxed{y = 0 \text{ or } y = \pm\frac{5}{2}}$

6. (10 points) One number is five more than another number. The product of the numbers is 84. Use the algebraic method to find all such numbers.
   SOLN: Let x = the smaller number. Then $x+5$ = the larger number and $x(x+5) = 84$ so $x^2 + 5x - 84 = 0 \Leftrightarrow (x-7)(x+12) = 0 \Leftrightarrow x = -12$ or $x = 7$ So the numbers could be $\boxed{-12 \text{ and } -7}$ or $\boxed{7 \text{ and } 12}$.

7. (15 points) A ball is thrown into the air with an upward velocity of 20 feet per second from a building 50 feet high. The equation for the height h of the ball above the ground at time t is $\boxed{h = 50 + 20t - 16t^2}$



   (a) What is the height of the ball at $t = 2$ seconds?
   SOLN: At $t = 2$, $h = 50 + 20(2) - 16(4) = 90 - 64 = 26$ feet.

   (b) Write an equation whose solution gives the time when the ball hits the ground.
   $h = 0 \Leftrightarrow 50 + 20t - 16t^2 = 0$

   (c) Find the time when the ball hits the ground.
   $16t^2 - 20t - 50 = 0 \Leftrightarrow 2(8t^2 - 10t - 25) = 0$
   $\Leftrightarrow 2(8t^2 + 10t - 20t - 25) = 0 \Leftrightarrow 2(2t(4t+5) - 5(4t-5)) = 0$
   $\Leftrightarrow 2(2t-5)(4t+5) = 0 \Rightarrow \boxed{t = \frac{5}{2} = 2.5 \text{ seconds}}$