Math 54 - Fall 2014 - Exam 1 Solutions

1. Simplify (solution steps are in the boxes.)

   (a) $\dfrac{6x}{35} \cdot \dfrac{5x}{9} = \boxed{\dfrac{2x^2}{21}}$

   (b) $\dfrac{3(-7) + 8(7)}{-4^2 - 4} = \boxed{\dfrac{-21 + 56}{-16 - 4} = \dfrac{35}{-20} = -\dfrac{5}{4}}$

   (c) $\dfrac{1}{6}(9x - 4) - \dfrac{1}{5}(x - 1) = \boxed{\dfrac{3}{2}x - \dfrac{2}{3} - \dfrac{1}{5}x + \dfrac{1}{5} = \left(\dfrac{3}{2} - \dfrac{1}{5}\right)x - \dfrac{2}{3} + \dfrac{1}{5} = \dfrac{13}{10}x - \dfrac{7}{15}}$
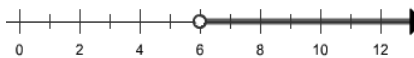
2. Solve each equation.

   (a) $\dfrac{3}{7}x = -6 \boxed{\Leftrightarrow x = -6 \cdot \dfrac{7}{3} \Leftrightarrow x = -14}$

   (b) $\dfrac{1}{3}(3x - 2) = \dfrac{1}{4}(8x - 1) \boxed{\Leftrightarrow 12 \cdot \dfrac{1}{3}(3x - 2) = 12 \cdot \dfrac{1}{4}(8x - 1) \Leftrightarrow 4(3x - 2) = 3(8x - 1)}$

   $\boxed{\Leftrightarrow 12x - 8 = 24x - 3 \Leftrightarrow -12x = 5 \Leftrightarrow x = -\dfrac{5}{12}}$

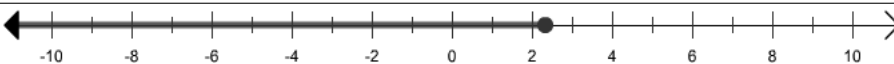   (c) $1 + 2(4x - 3y) = 0 \boxed{\Leftrightarrow 1 + 8x - 6y = 0 \Leftrightarrow x = \dfrac{6y - 1}{8} \Leftrightarrow y = \dfrac{8x + 1}{6}}$

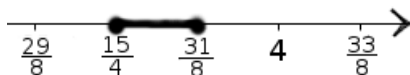3. Solve each inequality and graph the solution on the number line.

   (a) $-\dfrac{5}{3}x < -10 \boxed{x > -\dfrac{3}{5}(-10) \Leftrightarrow x > 6}$

   (b) $\dfrac{2}{5}(2x - 3) \le \dfrac{1}{2}(x - 1) \boxed{\Leftrightarrow 10 \cdot \dfrac{2}{5}(2x - 3) \le 10 \cdot \dfrac{1}{2}(x - 1) \Leftrightarrow 4(2x - 3) \le 5(x - 1) \Leftrightarrow 8x - 12 \le 5x - 5}$

   $\boxed{\Leftrightarrow 3x \le 7 \Leftrightarrow x \le \dfrac{7}{3}}$

   (c) $1 \le 2(2x - 7) \le \dfrac{3}{2} \boxed{2 \le (8x - 28) \le 3 \Leftrightarrow 30 \le 8x \le 31 \Leftrightarrow \dfrac{15}{4} \le x \le \dfrac{31}{8}}$

4. Juan's age is three times Sue's age now. In 12 years, Juan will be 24 years older than Sue. Answer the questions below to use the algebraic method to find their ages now.

   (a) $\boxed{\text{Let } x = \text{Sue's age now.}}$

   (b) $\boxed{\text{Then Juan's age is } 3x}$

   (c) $\boxed{\text{In 12 years Sue will be } x + 12 \text{ and Juan will be } 3x + 12}$.

   (d) Write an equation involving the unknown. $\boxed{3x + 12 = (x + 12) + 24}$

(e) Solve the equation and check your answer. $\boxed{2x = 24 \Leftrightarrow x = 12}$
Sue is 12 years old now, and Juan is 36. Of course, Juan will always be 24 years older than Sue.

5. A woman has money in two accounts. One account pays 8% annual interest while the other pays 10% annual interest. She has $400 more invested at 10% than she has at 8% and her total interest for the year is $260. Answer the questions below to use the algebraic method to find how much does she has in each account.

   (a) Identify your unknown as the amount invested in one account or the other.
   $\boxed{\text{Let } x = \text{the amount invested at } 8\%.}$

   (b) Write the amount invested in the other account in terms of your unknown.
   $\boxed{\text{Then } x + 400 = \text{the amount invested in the account with } 10\% \text{ interest.}}$

   (c) Write the interest earned on each account in terms of the unknown.
   $\boxed{\text{The } 8\% \text{interest on } x \text{ is } 0.08x \text{ and the interest earned at } 10\% \text{ is } 0.1(x + 400)}$

   (d) Write an equation involving the unknown. $\boxed{0.08x + 0.1(x + 400) = 260}$

   (e) Solve the equation and check your answer. $\boxed{0.18x + 40 = 260 \Leftrightarrow 0.18x = 220 \Leftrightarrow x = \dfrac{220}{0.18} = \dfrac{11000}{9}}$

   $\boxed{\dfrac{11000}{9} \approx \$1222.22 \text{ was invested at } 8\% \text{ so } \$1622.22 \text{ was invested at } 10\%. \text{ To check this,}}$

   $\boxed{\text{observe that } 8\% \text{ of } \$1222.22 \text{ is } \approx \$97.78 \text{ and } 10\% \text{ of } \$1622.22 \text{ is } \approx \$162.22,}$

   $\boxed{\text{totalling } \$97.78 + \$162.22 = \$260}$

6. Joann has a collection of dimes and quarters worth $3.95. She has 6 more quarters than dimes. Use the algebraic method to determine the number of dimes and the number of quarters in the collection.

   Let $x =$ the *number* of dimes. Then $x + 6 =$ the *number* of quarters. The value of the dimes is $10x$ and the value of the quarters is $25(x + 6)$. Thus the total value is $10x + 25(x + 6) = 395$ (in cents). Solving the equation yields $35x + 150 = 395 \Leftrightarrow 35x = 245 \Leftrightarrow x = 7$
   . So there are 7 dimes and 13 quarters. This works out since $10 \cdot 7 + 25 \cdot 13 = 70 + 325 = 395$.

7. Write and inequality whose solution is the given graph.

   (a)   ans: $-4 < x \leq 3$.

   (b)   ans: $x \leq 4$ or $7 \leq x$