Math 40 – Exercise 4.2#6:

Let t = the number of years since 1990 and s = sales of something or other in that year.

The following data are tabulated:

t	0	1	2	3	4
S	54	82	194	446	726

Our task is to find an equation for the parabola that fits the data for t = 1, 2, 3 and then see how well that fits the other two data.

We seek a, b and c so that $S = at^2 + bt + c$ fits the middle three points:

Rewrite the equation in the equivalent form $t^2a + tb + c = S$ and plug in the data pairs to produce the 3X3 linear system in a, b and c:

$$a + b + c = 82$$
$$4a + 2b + c = 194$$
$$9a + 3b + c = 446$$

Eliminating c from first the first and second equations and then from the first and third equations, we have

$$3a + b = 102$$
$$8a + 2b = 364$$

This is easily solved: $a = 70$ and $b = -98$.

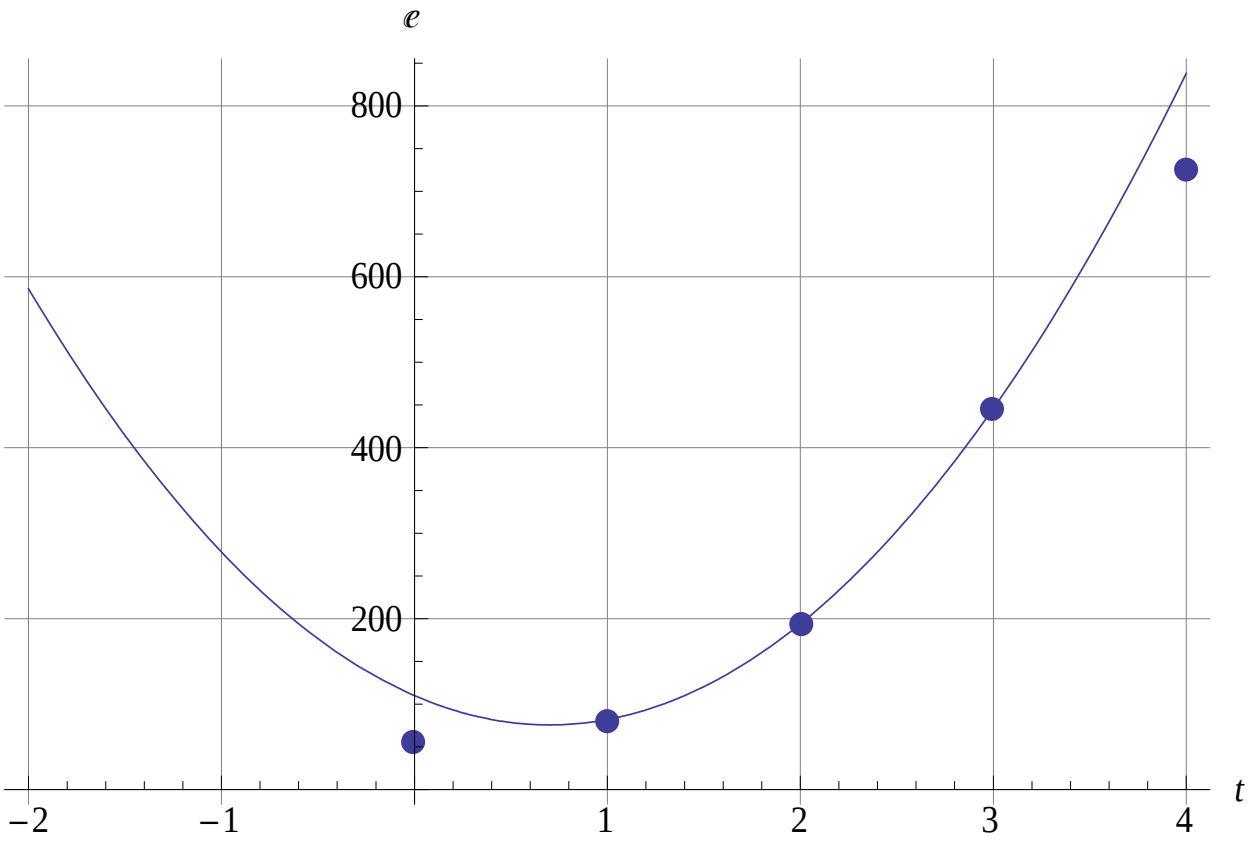
Plug back into $a + b + c = 82$ and deduce $c = 110$.

Having the parameter values for a, b, and c allows us to, somewhat gratuitously, compute
$h = -b/2a = 98/140 = 49/70 = 7/10$ and
$k = c - b^2/(4a) = 110 - 9604/280 = 110 - 2401/70 = (1100 - 343)/10 = 75.7$

Thus $S = 70t^2 - 98t + 110 = 70\left(t - 0.7\right)^2 + 75.7$ Factored form would easy now since the vertex form is easily solved for the zeros. But that might be overly gratuitous!

Note that $S(0) = 110$ is pretty far off the tabulated value of 54 and
$S(4) = 70(3.3)^2+75.7 = 70(10.89)+75.7 = 762.3+75.7 = 838$ is a bit more than the tabulated value of 726.

A visualization of the parabola and the data follows: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