Math 40 - Chapters 6 and 7 Test – Spring '10 Name: ______________________

Directions: Show all work for credit. Write all responses on separate paper.

1. Simplify each of the following:

a. $\left(\frac{5}{2}\right)^{-4}$ b. $\left(-\frac{27}{32}\right)^{4/3}$ c. $\left(\frac{27}{125}\right)^{-3/4}$

2. Simplify: $\left(-\frac{A^{-3}}{2B^2}\right)^4\left(\frac{3B^3}{A^{-6}}\right)^3$.

3. If $G(u)=\left(\frac{u+3}{2}\right)^{-2/3}$, find u so that $G(u)=\frac{1}{4}$.

4. Find a power function (of the form, $f(x)=kx^p$) that is consistent with the table below.

x	$\frac{5}{16}$	$\frac{5}{8}$	1	$\frac{5}{2}$
$f(x)$	$\frac{16}{5}$	$\frac{4}{5}$	$\frac{5}{16}$	$\frac{1}{20}$

5. Write an equation for the circle in the *x*-*y* plane with a diameter from (3,6) to (3,10).

6. Find the center and radius of the circle described by the equation $x^2-2x+y^2+8y=26$

7. The line segment from (0,0) to (11,5) is the diameter of a circle. Find an equation for the line tangent to the circle at (0,0).

8. Write an equivalent expression without a radical or rational exponent in the denominator:

a. $\frac{1}{3-\sqrt{x}}$ b. $\frac{1}{\sqrt[3]{4x^5}}$

9. Use the method of equating squares to find both solutions to the equation: $\sqrt{7x+11}-\sqrt{5x-1}=2$.
Hint: Isolate a square root and equate squares twice.

10. The National Marine Fisheries Service has concluded that the population of sockeye salmon at Ozette Lake has been declining from a population of about 1000 since 1977 to the point where only about 600 fish returned to spawn in 1997. Let *t* represent the number of years since 1977.
    a. Write a linear function (in the form $P = mt + b$) modeling the population over this 20-year period. You'll need to figure out the slope, *m* and the *y*-intercept, *b*.
    b. In what year does the linear model predict the population will be down to one fish?
    c. Write an exponential function (in the form $P = P_0(1+r)^t$)
    d. In what year does the exponential model predict the population will be down to one fish? (you will need a logarithm to solve this?)

11. Find an exponential function (of the form $y = P(1+r)^x$) through points (x, y) = (0,2) and (3,5).

12. Solve for the unknown (remember to show steps):

a. Solve for *y*: $\log_3 y = -5$. b. Solve for *k*: $20\left(1+\frac{0.06}{24}\right)^{24k}=400$.

Solutions For Chapter 6 and 7 Test

1. Simplify each of the following:
   a. $\left(\frac{5}{2}\right)^{-4}=\frac{2^4}{5^4}=\frac{16}{625}$
   b. $\left(-\frac{27}{32}\right)^{4/3}=\left(-\frac{3^3}{2^5}\right)^{4/3}=-\frac{\left(3^3\right)^{4/3}}{\left(2^5\right)^{4/3}}=-\frac{3^4}{2^{20/3}}\frac{2^{1/3}}{2^{1/3}}=\frac{81\sqrt[3]{2}}{128}$
   c. $\left(\frac{27}{125}\right)^{-3/4}=\left(\frac{3^3}{5^3}\right)^{-3/4}=\frac{5^{9/4}}{3^{9/4}}=\frac{25\sqrt[4]{5}}{9\sqrt[4]{3}}\frac{\sqrt[4]{27}}{\sqrt[4]{27}}=\frac{25\sqrt[4]{135}}{27}$

2. Simplify: $\left(-\frac{A^{-3}}{2B^2}\right)^4\left(\frac{3B^3}{A^{-6}}\right)^3=\frac{A^{-12}\left(27B^9\right)}{16B^8A^{-18}}=\frac{27A^6B}{16}$.

3. If $G(u)=\left(\frac{u+3}{2}\right)^{-2/3}$, find u so that $G(u)=\frac{1}{4}$.

   $\left(\frac{u+3}{2}\right)^{-2/3}=\frac{1}{4}\Leftrightarrow\frac{u+3}{2}=\left(\frac{1}{4}\right)^{-3/2}\Leftrightarrow\frac{u+3}{2}=8\Leftrightarrow u=13$

4. Find a power function (of the form, $f(x)=kx^p$) that is consistent with the table below.

x	$\frac{5}{16}$	$\frac{5}{8}$	1	$\frac{5}{2}$
$f(x)$	$\frac{16}{5}$	$\frac{4}{5}$	$\frac{5}{16}$	$\frac{1}{20}$

SOLN: Note that $f(1)=\frac{5}{16}\Rightarrow k=\frac{5}{16}$. We can then use any one of the remaining in/out pairs to find p.

$$f\left(\frac{5}{16}\right)=\frac{5}{16}\left(\frac{5}{16}\right)^p=\frac{16}{5}\Leftrightarrow\left(\frac{5}{16}\right)^p=\left(\frac{16}{5}\right)^2\Rightarrow p=-2$$

5. Write an equation for the circle in the *x*-*y* plane with a diameter from (3,6) to (3,10).
   SOLN: Clearly the center of the circle is at (3,8) and the radius is 2. So the equation is
   $(x-3)^2+(y-8)^2=4$

6. Find the center and radius of the circle described by the equation $x^2-2x+y^2+8y=26$

   SOLN: $x^2-2x+y^2+8y=26\Leftrightarrow x^2-2x+1+y^2+8y+16=26+1+16$
   $\Leftrightarrow(x-1)^2+(y+4)^2=43$

   So the center is (1,–4) and the radius is $\sqrt{43}$.

7. The line segment from (0,0) to (11,5) is the diameter of a circle. Find an equation for the line tangent to the circle at (0,0).

   SOLN: The center is (11/2,5/2) and the radius is $\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{11}{2}\right)^2}=\sqrt{\frac{25+121}{4}}=\frac{\sqrt{146}}{2}$, but we don't need to know those things. It is sufficient that the tangent line is perpendicular to the radius line at that point. The slope of the diameter is 11/5, so the tangent line has slope –5/11 and passes through (0,0), so the equation is $y=-\frac{5}{11}x$

8. Write an equivalent expression without a radical or rational exponent in the denominator:

a. $\dfrac{1}{\left(3-\sqrt{x}\right)}\dfrac{\left(3+\sqrt{x}\right)}{\left(3+\sqrt{x}\right)}=\dfrac{3+\sqrt{x}}{9-x}$ b. $\dfrac{1}{\sqrt[3]{4x^5}}\dfrac{\sqrt[3]{2x}}{\sqrt[3]{2x}}=\dfrac{\sqrt[3]{2x}}{2x^2}$

9. Use the method of equating squares to find both solutions to the equation: $\sqrt{7x+11}-\sqrt{5x-1}=2$.
Hint: Isolate a square root and equate squares twice.

SOLN:

$$\left(\sqrt{7x+11}\right)^2=\left(2+\sqrt{5x-1}\right)^2 \Leftrightarrow 7x+11=4+4\sqrt{5x-1}+5x-1$$

$$\Leftrightarrow 2x+8=4\sqrt{5x-1} \Rightarrow \left(x+4\right)^2=4\left(5x-1\right)$$

$$\Leftrightarrow x^2-12x+20 \Leftrightarrow \left(x-10\right)\left(x-2\right)=0 \Leftrightarrow \boxed{x=10} \text{ or } \boxed{x=2}$$

10. The National Marine Fisheries Service has concluded that the population of sockeye salmon at Ozette Lake has been declining from a population of about 1000 since 1977 to the point where only about 600 fish returned to spawn in 1997. Let t represent the number of years since 1977.

a. Write a linear function (in the form $P = mt + b$) modeling the population over this 20-year period. You'll need to figure out the slope, m and the y-intercept, b.

SOLN: The slope is $\dfrac{600-1000}{20-0}=-20$ so the linear model is $P=1000-20t$

b. In what year does the linear model predict the population will be down to one fish?

SOLN: The one fish population will occur when $1000-20t=1 \Leftrightarrow t=\dfrac{999}{20}=49.95$ or just at the end of 2026.

c. Write an exponential function (in the form $P = P_0(1 + r)^t$)

SOLN: Clearly the initial population is $P_0 = 1000$. So, after 20 years we have $1000(1 + r)^{20} = 600$ or $(1 + r)^{20} = 0.6$ so $1+r=0.6^{1/20}\approx 0.975$ so $r \sim -0.025$

d. In what year does the exponential model predict the population will be down to one fish? (you will need a logarithm to solve this?)

SOLN: $1000\left(0.975\right)^t=1 \Leftrightarrow \left(0.975\right)^t=0.001 \Leftrightarrow t=\log_{0.975}\left(0.001\right)=\dfrac{\log 0.001}{\log 0.975}\approx 272.8$

So that would be toward the end of the year 2249.

11. Find an exponential function (of the form $y = P(1 + r)^x$) through points $(x, y) = (0,2)$ and $(3,5)$.
SOLN: $y = 2(1 + r)^x$ so $5 = 2(1 + r)^3$ so $(1 + r)^3 = 2.5$ and $1 + r = 2.5^{1/3} \sim 1.3572$ and $y = 2(1.3572)^x$.

12. Solve for the unknown (remember to show steps):

a. Solve for y: $\log_3 y=-5 \Leftrightarrow y=3^{-5}=\dfrac{1}{243}$.

b. Solve for k:

$$20\left(1+\frac{0.06}{24}\right)^{24k}=400 \Leftrightarrow \left(1+\frac{0.01}{4}\right)^{24k}=20 \Leftrightarrow \left(1.0025\right)^{24k}=20 \Leftrightarrow 24k=\log_{1.0025}20.$$

$$\Leftrightarrow 24k=\frac{\log 20}{\log 1.0025} \Leftrightarrow k=\frac{\log 20}{24\log 1.0025}\approx 49.99$$