Math 40 – Chapter 4 Test – Spring ‘10 Name: ______________________

Directions: Write all responses on separate paper. Show all work for credit.

1. Write equations for the parabola whose graph is shown in
   a. Factored form, specifying values for a, r_1, and r_2 in the equation,

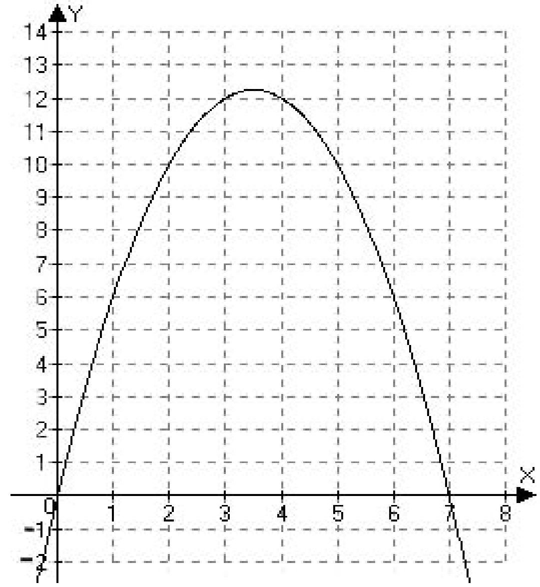
      $$y = a(x - r_1)(x - r_2).$$

   b. Vertex form, specifying the values of a, h, and k in the equation,

      $$y = a(x - h)^2 + k.$$

   c. Descending powers form, specifying the values of a, b, and c in the equation

      $$y = ax^2 + bx + c.$$



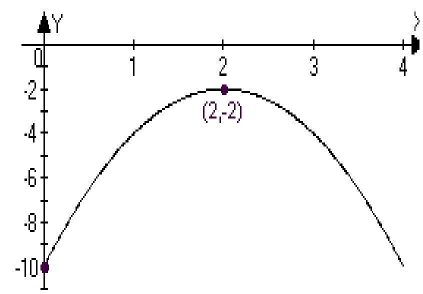
2. Write an equation for the parabola with intercepts at $(2,0),(0,1)$ and $\left(-\frac{5}{2},0\right)$

3. Write equations for the parabola whose graph is shown below
   a. Vertex form, specifying the values of a, h, and k in the equation,

      $$y = a(x - h)^2 + k.$$

   b. Descending powers form, specifying the values of a, b, and c in the equation

      $$y = ax^2 + bx + c.$$



4. Write a quadratic inequality whose solutions is given in interval notation:

   a. $x \in \left(-\infty, -\frac{5}{3}\right] \cup [2, \infty)$ b. $x \in \left[-\frac{5}{3}, 2\right]$

For problems 5-7, find the coordinates of the (a) x-intercepts (b) the y-intercept and (c) the vertex of the parabola whose equation is given, then carefully construct a graph showing these features.

5. $y = 4x^2 - 36$

6. $y = -(x-2)(x+7)$

7. $y = -2(x-5)^2 + 18$

8. Solve the system of equations algebraically and verify your solutions with a graph.

$$y = 32 - 2(x-4)^2$$
$$y = 4x + 10$$

9. Solve the inequality. Write the solution in interval notation.

   a. $-\frac{7}{11}(x-1)(x-12) \geq 0$

   b. Solve the inequality: $27(x-2)^2 - 12 \geq 0$.

10. Find an equation for the parabola passing the points (1,1), (2,–3/2) and (3,–5). That is, find a, b, and c so that $y = ax^2 + bx + c$.

Math 40 – Chapter 4 Test Solutions – Spring '10

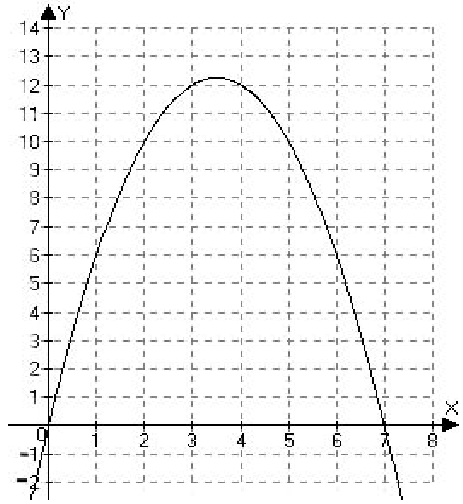
1. Write equations for the parabola whose graph is shown in
   a. Factored form, specifying values for a, r_1, and r_2 in the equation, $y = a(x - r_1)(x - r_2)$.

      SOLN: $y = a(x-0)(x-7) = ax(x-7)$. The parabola appears to go through (3,12) so that 12 = –12*a* so that *a* = –1 and $\boxed{y = -x(x-7)}$ is factored form.

   b. Vertex form, specifying the values of a, h, and k in the equation, $y = a(x-h)^2 + k$.

      $$y = -x(x-7) = -x^2 + 7x - \frac{49}{4} + \frac{49}{4} = \boxed{y = -\left(x - \frac{7}{2}\right)^2 + \frac{49}{4}}$$

   c. Descending powers form, specifying the values of *a*, *b*, and *c* in the equation is simple now: $\boxed{y = -x^2 + 7x}$



2. Write an equation for the parabola with intercepts at $(2,0),(0,1)$ and $\left(-\frac{5}{2}, 0\right)$

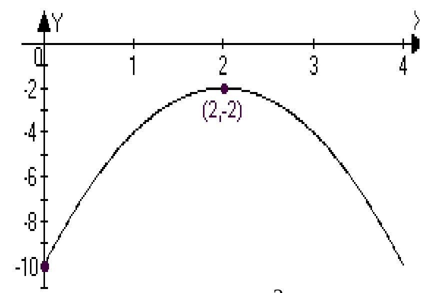
   SOLN: From the two intercepts, $y = a(x-2)(2x+5)$. We can use the *y*-intercept to find *a*:

   $$1 = a(-2)(5) \Rightarrow a = \frac{-1}{10} \text{ so that } \boxed{y = -\frac{1}{10}(x-2)(2x+5) = -\frac{1}{5}x^2 - \frac{1}{10}x + 1 = -\frac{1}{5}\left(x + \frac{1}{4}\right)^2 + \frac{81}{80}}$$

3. Write equations for the parabola whose graph is shown below
   a. Vertex form, specifying the values of a, h, and k in the equation, $y = a(x-h)^2 + k$.

      SOLN: Plugging in the coordinates of the vertex in vertex form: $y = a(x-2)^2 - 2$. To solve for *a* use the *y*-intercept:

      $-10 = a(-2)^2 - 2 \Leftrightarrow a = -2$ so $\boxed{y = -2(x-2)^2 - 2}$



   b. Descending powers form, specifying the values of *a*, *b*, and *c* in the equation $y = ax^2 + bx + c$.

      SOLN: $y = -2(x^2 - 4x + 4) - 2 \Rightarrow \boxed{y = -2x^2 + 8x - 10}$

4. Write a quadratic inequality whose solutions is given in interval notation:
   a. $x \in \left(-\infty, -\frac{5}{3}\right] \cup [2, \infty)$

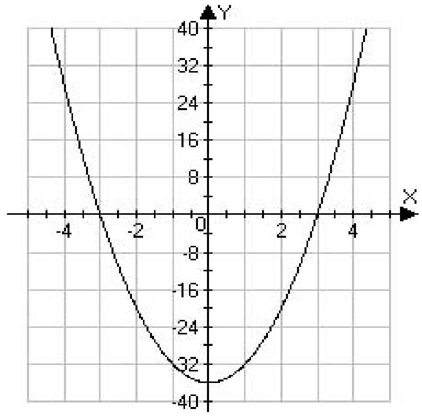
      SOLN: $(3x+5)(x-2) \ge 0$

   b. $x \in \left[-\frac{5}{3}, 2\right]$

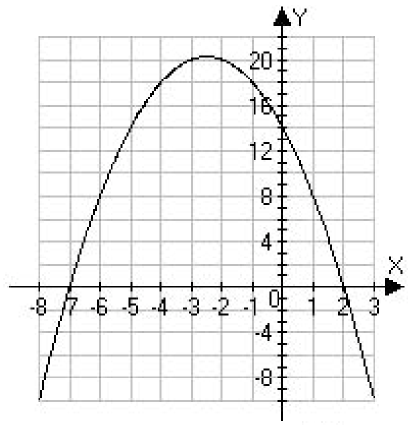
      SOLN: $(3x+5)(x-2) \le 0$

For problems 5-7, find the coordinates of the (a) *x*-intercepts (b) the *y*-intercept and (c) the vertex of the parabola whose equation is given, then carefully construct a graph showing these features.

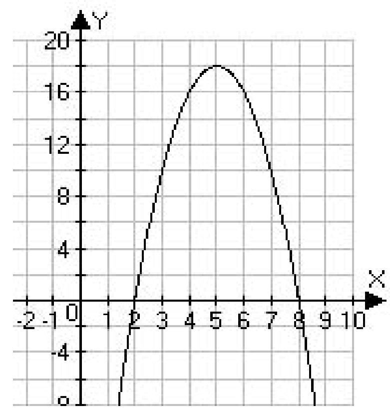
5. $y = 4x^2 - 36$
$= 4(x-3)(x+3)$



6. $y = -(x-2)(x+7)$
$= \frac{81}{4} - \left(x + \frac{5}{2}\right)^2$



7. $y = -2(x-5)^2 + 18$
$= -2x^2 + 20x - 32$
$= -2(x-8)(x-2)$



8. Solve the system of equations algebraically and verify your solutions with a graph.

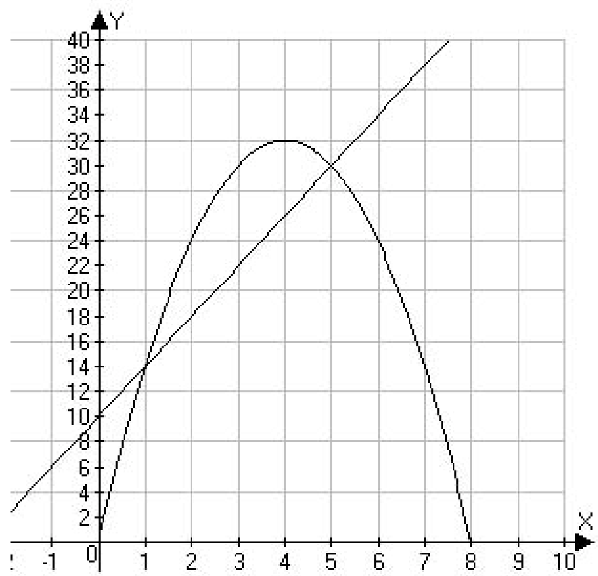
$y = 32 - 2(x-4)^2$

$y = 4x + 10$

SOLN:

$y = 32 - 2(x-4)^2 = -2x^2 + 16x = -2x(x-8)$ describes a parabola with vertex (4,32) and intercepts at (0,0) and (8,0).
The other equation describes a line with slope *m* = 4 and *y*-intercept (0,10), as shown in the diagram at right.From the graph it appears the intersections are at (1,14) and (5,30).
Solve the following equivalent equations:

$32 - 2(x-4)^2 = 4x + 10 \Leftrightarrow (x-1)(x-5) = 0$



9. Solve the inequality. Write the solution in interval notation.

a. $-\frac{7}{11}(x-1)(x-12) \geq 0$

SOLN: $-\frac{7}{11}(x-1)(x-12) \geq 0$ has boundary points at *x* = 1 and *x* = 12. A quick check shows that *x* = 0 does not solve the inequality, so the solution is between the boundaries: $\boxed{x \in [1,12]}$

b. Solve the inequality: $27(x-2)^2 - 12 \geq 0$.

SOLN:

$$27(x-2)^2 - 12 \geq 0 \Leftrightarrow (x-2)^2 \geq \frac{12}{27} = \frac{4}{9} \Leftrightarrow \sqrt{(x-2)^2} \geq \frac{2}{3}$$

$$\Leftrightarrow x - 2 \geq \frac{2}{3} \text{ or } x - 2 \leq -\frac{2}{3} \Leftrightarrow \boxed{x \in \left(-\infty, \frac{4}{3}\right] \cup \left[\frac{8}{3}, \infty\right)}$$

10. Find an equation for the parabola passing the points (1,1), (2,–3/2) and (3,–5).
That is, find *a*, *b*, and *c* so that $y = ax^2 + bx + c$.

SOLN: We plug the three given (x,y) pairs into $x^2a + xb + c = y$ to get a 3X3 linear system:

$$a+b+c=1$$
$$4a+2b+c=-\frac{3}{2}$$
$$9a+3b+c=-5$$

The difference of the first two equations is

$$3a+b=-\frac{5}{2}$$

and the difference of the second and third equations is

$$5a+b=-\frac{7}{2}$$

Subtracting the first of these from the second yields

$$2a=-1 \Leftrightarrow \boxed{a=-\frac{1}{2}}$$

And substituting this back into the first of these yields

$$-\frac{3}{2}+b=-\frac{5}{2} \Leftrightarrow \boxed{b=-1}$$

And since the sum of a, b and c must be 1, $\boxed{c=\frac{5}{2}}$

Putting these together we have $y=-\frac{1}{2}x^2-x+\frac{5}{2}=\frac{-1}{2}(x+1)^2+3=-\frac{1}{2}\left(x+1-\sqrt{6}\right)\left(x+1+\sqrt{6}\right)$

So that the parabola has vertex (–1,3) and intercepts $\left(-1-\sqrt{6},0\right),\left(0,\frac{5}{2}\right),\left(-1+\sqrt{6},0\right)$: