Math 40 – Chapter 3 Test – Spring 10 Name: ______________________________

Directions: Show all work for credit. Write all responses on separate paper. Don't use a calculator.

1. Solve by extracting roots:
   a. $9(x+5)^2 = 1$
   b. $2\left(x-\frac{1}{2}\right)^2 = \frac{3}{2}$

2. Solve by factoring:
   a. $x^2 - x = 30$
   b. $2x^2 - 7x + 3 = 0$

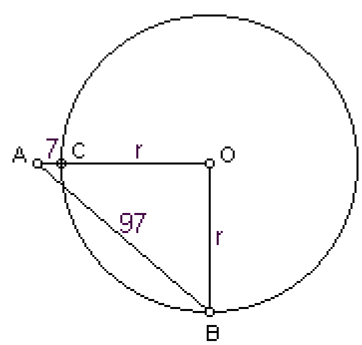
3. Solve by completing the square:
   a. $x^2 + 10x - 2 = 0$
   b. $2x^2 + 8x - 6 = 0$

4. Suppose a quadratic equation has solutions $x = \frac{1}{4}$ and $x = -\frac{3}{5}$
   a. Use the zero product principle to write the equation in the form $(x - r_1)(x - r_2) = 0$.
   b. Find integer values of *a*, *b* and *c* so that this equation is equivalent to $ax^2 + bx + c = 0$

5. A stone is thrown upward so that its height *h* after *t* seconds is given by $h = -16t^2 + 56t + 6$, where *h* is measured in feet.
   a. At what time, *t*, will the stone reach its maximum height?
   b. What is this maximum height?

6. Find solutions for $8x^2 = 49$. Write the solutions in simplest radical form.

7. Solve $r = bx - ax^2$ for *x* in terms of *a*, *b* and *r*.



8. In the diagram at right, ΔAOB is a right triangle with right angle at the circle's center, *O*. Find the radius *r* of the circle if *AC* = 7 and *AB* = 97.

9. If the longer leg of a right triangle is 1 foot less than four times the short leg and the hypotenuse is 1 foot longer than four times the shorter leg, what are the lengths of the legs?

10. Make a table of values including the vertex and the intercepts for the parabola whose equation is $y = 4 - x^2$ and sketch a graph for the parabola showing these features.

11. If the long leg of a right triangle is 11 feet less than twice the short leg and the hypotenuse is 3 feet less than twice the short leg, what are the lengths of the legs?

12. The hypotenuse of a right triangle is 10 more than three times the short leg. If the square of the longer leg is 912, what is the length of the hypotenuse?

13. Use the quadratic formula to solve for *n* in terms of *x*: $4n^2 + 8nx + 3x^2 = 0$.

14. Consider the equation $x^2 + 3x + 5 = 0$.

    a. What is the discriminant of this quadratic?
    b. Where is the vertex of $y = x^2 + 3x + 5$?
    c. Which way does the parabola comprised by the solution set of $y = x^2 + 3x + 5$ open? How do you know?
    d. Make a table of values including the vertex, the y-intercept and the point which is the reflection of the y-intercept across the line of symmetry for this parabola.

Math 40 – Chapter 3 Test Solutions – Spring '10

1. Solve by extracting roots:
   a. $9(x+5)^2=1$

      SOLN: $9(x+5)^2=1 \Leftrightarrow (x+5)^2=\frac{1}{9} \Leftrightarrow x+5=\pm\frac{1}{3} \Leftrightarrow x=-5\pm\frac{1}{3}=-\frac{16}{3}$ or $-\frac{14}{3}$

   b. $2\left(x-\frac{1}{2}\right)^2=\frac{3}{2}$

      SOLN: $2\left(x-\frac{1}{2}\right)^2=\frac{3}{2} \Leftrightarrow \left(x-\frac{1}{2}\right)^2=\frac{3}{4} \Leftrightarrow x-\frac{1}{2}=\pm\frac{\sqrt{3}}{2} \Leftrightarrow \boxed{x=\frac{1}{2}\pm\frac{\sqrt{3}}{2}}$

2. Solve by factoring:
   a. $x^2-x=30$

      SOLN: $x^2-x=30 \Leftrightarrow x^2-x-30=0 \Leftrightarrow (x-6)(x+5)=0 \Leftrightarrow \boxed{x=6 \text{ or } x=-5}$

   b. $2x^2-7x+3=0$

      SOLN: $2x^2-7x+3=0 \Leftrightarrow (2x-1)(x-3)=0 \Leftrightarrow \boxed{x=\frac{1}{2} \text{ or } x=3}$

3. Solve by completing the square:
   a. $x^2+10x-2=0$

      SOLN: $x^2+10x-2=0 \Leftrightarrow x^2+10x=2 \Leftrightarrow x^2+10x+25=2+25 \Leftrightarrow (x+5)^2=27$

      $\Leftrightarrow x+5=\pm\sqrt{27} \Leftrightarrow \boxed{x=-5\pm 3\sqrt{3}}$

   b. $2x^2+8x-6=0$

      SOLN: $x^2+4x=3 \Leftrightarrow x^2+4x+4=7 \Leftrightarrow (x+2)^2=7 \Leftrightarrow x+2=\pm\sqrt{7} \Leftrightarrow \boxed{x=-2\pm\sqrt{7}}$

4. Suppose a quadratic equation has solutions $x=\frac{1}{4}$ and $x=-\frac{3}{5}$
   a. Use the zero product principle to write the equation in the form $(x-r_1)(x-r_2)=0$.

      SOLN: $\left(x-\frac{1}{4}\right)\left(x+\frac{3}{5}\right)=0$

   b. Find integer values of *a*, *b* and *c* so that this equation is equivalent to $ax^2+bx+c=0$

      SOLN: $20\left(x-\frac{1}{4}\right)\left(x+\frac{3}{5}\right)=0\cdot 20 \Leftrightarrow (4x-1)(5x+3)=0 \Leftrightarrow \boxed{20x^2+7x-3=0}$

5. A stone is thrown upward so that its height *h* after *t* seconds is given by $h=-16t^2+56t+6$, where *h* is measured in feet.
   a. At what time, *t*, will the stone reach its maximum height?

      SOLN:

      $$h=-16t^2+56t+6=-16\left(t^2-\frac{7}{2}t\right)+6=-16\left(t^2-\frac{7}{2}t+\left(\frac{7}{4}\right)^2\right)+6+16\left(\frac{7}{4}\right)^2=-16\left(t-\frac{7}{4}\right)^2+55$$

      So the vertex is at (7/4,55) and the maximum height of 55 is reached after 7/4 seconds.

   b. What is this maximum height? (see above)

6. Find solutions for $8x^2 = 49$. Write the solutions in simplest radical form.

SOLN: $8x^2 = 49 \Leftrightarrow x^2 = \frac{49}{8} \Leftrightarrow x = \pm\sqrt{\frac{49}{8}} = \pm\frac{7}{\sqrt{8}} = \pm\frac{7}{2\sqrt{2}} = \boxed{\pm\frac{7\sqrt{2}}{4}}$

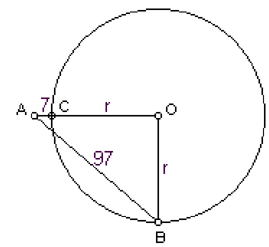
7. Solve $r = bx - ax^2$ for x in terms of a, b and r.

$$ax^2 - bx + r = 0 \Leftrightarrow x^2 - \frac{b}{a}x = -\frac{r}{a} \Leftrightarrow x^2 - \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{r}{a} + \left(\frac{b}{2a}\right)^2$$

SOLN: $$\Leftrightarrow \left(x - \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{r}{a} \Leftrightarrow x - \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{r}{a}}$$

$$\Leftrightarrow \boxed{x = \frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{r}{a}}}$$

8. In the diagram at right, ΔAOB is a right triangle with right angle at the circle's center, O. Find the radius r of the circle if $AC = 7$ and $AB = 97$.



SOLN:

$$(r+7)^2 + r^2 = 97^2 \Leftrightarrow r^2 + 14r + 49 + r^2 = 9409 \Leftrightarrow 2r^2 + 14r - 9360 = 0$$

Reduce by a factor 2: $r^2 + 7r - 4680 = 0$ To factor, we'd need to number whose product is 4680 and differ by 7. Since the square root of 4680 is somewhere in the sixties, that's a good place to start. 4680 = 60*78. That's not it. How about 65*72? That'll do! So $r^2 + 7r - 4680 = (r-65)(r+72)$ and so the radius of the circle is 65.

Add 65, 72, 97 to your list of Pythagorean triples.

9. If the longer leg of a right triangle is 1 foot less than four times the short leg and the hypotenuse is 1 foot longer than four times the shorter leg, what are the lengths of the legs?
SOLN: Let x = the length of the short leg. Then the long leg is $4x - 1$ and the hypotenuse is $4x + 1$ so that, by Pythagoras' theorem, $x^2 + (4x - 1)^2 = (4x + 1)^2$. Expanding the squares, $x^2 + 16x^2 - 8x + 1 = 16x^2 + 8x + 1$. Combining like terms to standard form, $x^2 - 16x = 0$ which we can solve by factoring. Ruling out $x = 0$ as silly, we get $x = 16$, so the longer leg is 63 and the hypotenuse is 65. Add 16, 63, 65 to your list of Pythagorean triples.

10. Make a table of values including the vertex and the intercepts for the parabola whose equation is $y = 4 - x^2$ and sketch a graph for the parabola showing these features.

SOLN: The vertex at (0,4) is also the y-intercept. The x-intercepts are at (–2,0) and (2,0).

x	–2	–1	0	1	2
y	0	3	4	3	0