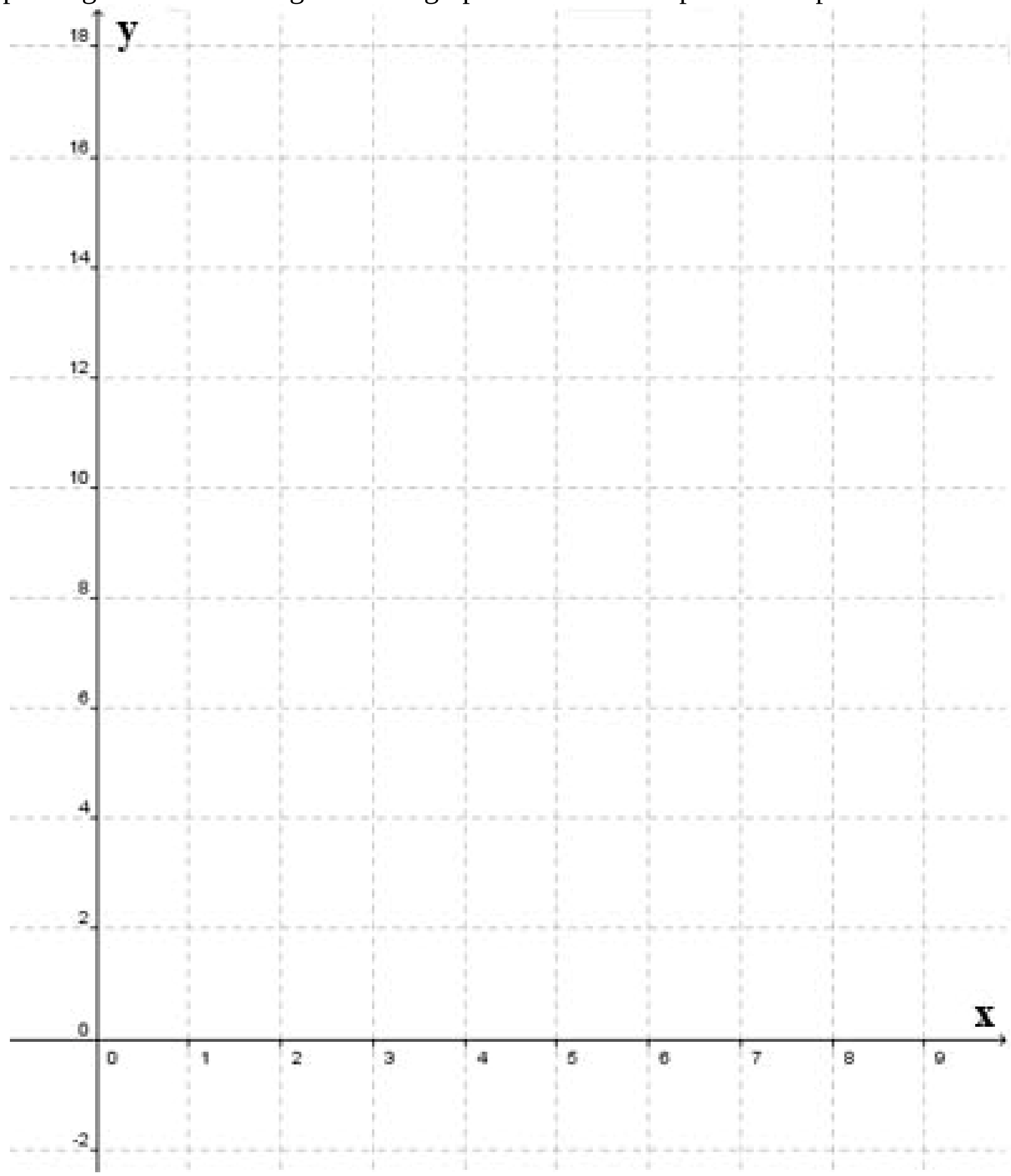
Math 40 – Intermediate Algebra - Chapter 2 Test   Name: ______________________

Directions:
Show all work.
Write all responses on separate paper, except the graph in #2.
Make large graphs and don't crowd your work into corners or margins.

1. Solve the system of equations by the method of graphing: $\begin{aligned} 2x + y &= 8 \\ x - y &= -5 \end{aligned}$

   Be sure to scale and label your axes and label the solution to the system with coordinates.

2. Solve the system: $\begin{aligned} 23x - 14y &= 17 \\ 2x + y &= 17 \end{aligned}$ and then illustrate how the two lines intersect at that point by plotting the two lines together in a graph. Tabulate and plot intercepts for each line.



3. Describe each of the following systems as either dependent, inconsistent, or independent and consistent. Show how you justify your conclusion.

a. $y = -\frac{17}{5}x$, $17x + 5y = 0$

b. $y = -\frac{17}{5}x$, $5x + 17y = 0$

c. $y = -\frac{17}{5}x + 1$, $17x + 5y = 1$

d. $y = \frac{17}{5}x$, $5x + 17y = 1$

4. Solve the system of equations by elimination, then illustrate in a graph how the two lines intersect at the solution point. Give coordinates of the solution.

$$\begin{aligned} 2x - 3y &= 10 \\ x - 2y &= 6 \end{aligned}$$

5. A concession stand sells apple juice and tangerine juice. The apple juice costs $1.50 per cup and the tangerine juice costs $0.85. If the concession stand made $88.80 of revenue selling 80 apple juice and tangerine juice cups during a day, how many tangerine juice cups were sold?

6. A collection includes nickels, dimes and quarters. There are 24 coins. The total value of the coins is $3 and there is one more nickel than there are dimes. Set up and solve a system of equations to find how many of each type of coin there are.

7. Solve the system by back substitution:

$$\begin{aligned} 3x - 4y + 5z &= 17 \\ 7y - 4z &= 16 \\ 3z &= 9 \end{aligned}$$

8. Find a solution the system (in *x*, *y* and *z*) with the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 4 & 8 & 8 \\ 3 & 0 & 0 & -12 \\ 0 & 5 & 7 & 17 \end{array}\right]$$

9. Solve the dependent system and express *x* and *y* in terms of *z*.

$$\begin{aligned} 2x - y + z &= 10 \\ x + 2y + 2z &= 5 \\ 8x + y + 7z &= 40 \end{aligned}$$

10. Graph the system of inequalities, label the vertices with coordinate values and shade the solution region:

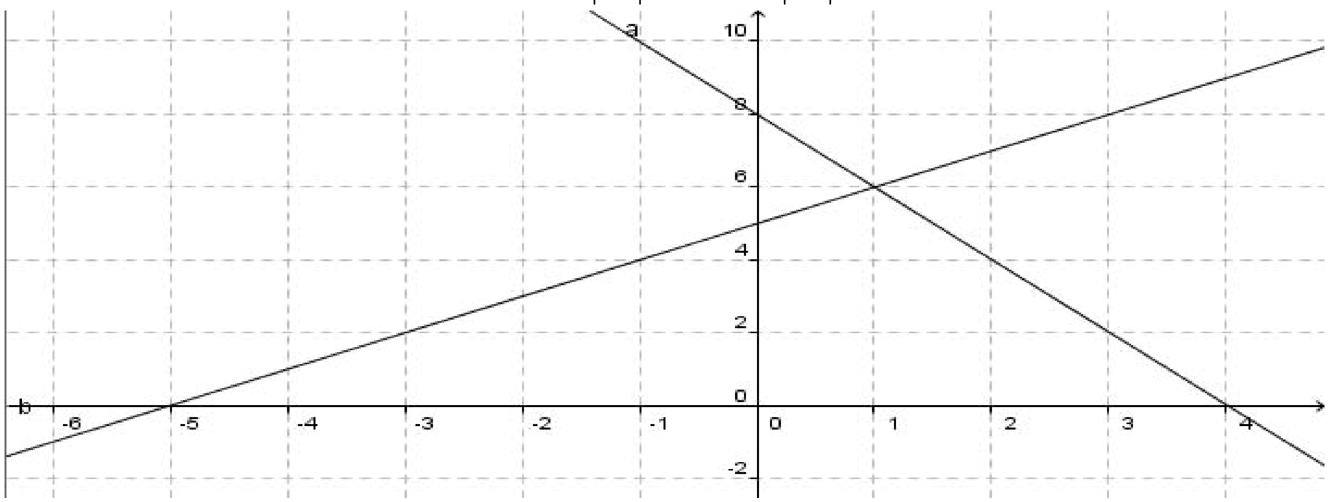
$$\begin{aligned} 3y - 4x &\le 9 \\ y + x &\le 10 \\ x \ge 0,\ \ y &\ge 0 \end{aligned}$$

Solutions For Chapter 2 Test Form A.

1. Solve the system of equations by the method of graphing: $\begin{aligned} 2x + y &= 8 \\ x - y &= -5 \end{aligned}$

   Be sure to scale and label your axes and label the solution to the system with coordinates.

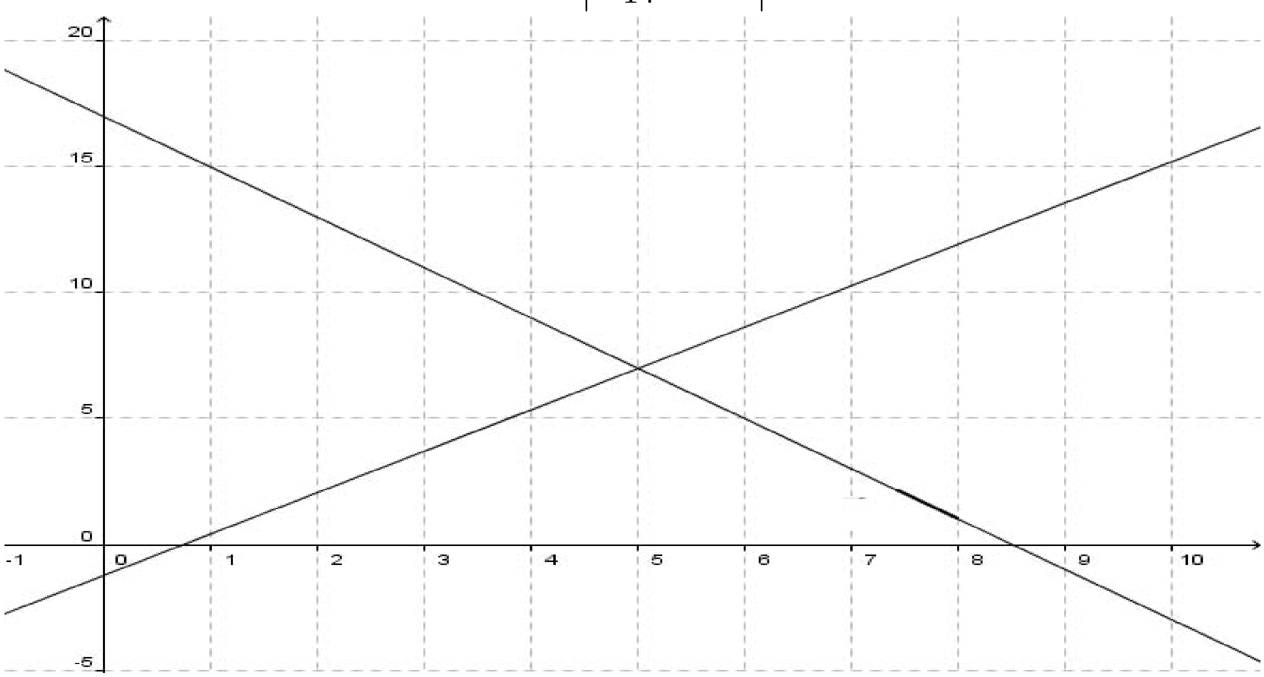
   SOLN: Intercepts tables are, respectively, $\begin{array}{c|c|c} x & 0 & 4 \\ \hline y & 8 & 0 \end{array}$ and $\begin{array}{c|c|c} x & 0 & -5 \\ \hline y & 5 & 0 \end{array}$. The intersection is at (1,6).



2. Solve the system: $\begin{aligned} 23x - 14y &= 17 \\ 2x + y &= 17 \end{aligned}$ and then illustrate how the two lines intersect at that point by plotting the two lines together in a graph. Tabulate and plot intercepts for each line.

   SOLN: Solving the second equation for y we get $y = 17 - 2x$ and substituting into the first yields $23x - 14(17 - 2x) = 17 \Leftrightarrow 51x - 238 = 17 \Leftrightarrow 51x = 255 \Leftrightarrow \boxed{x = 5}$ and so $\boxed{y = 7}$.

   Tabulating values for each graph we have $\begin{array}{c|c|c} x & 0 & \frac{17}{23} \approx 0.74 \\ \hline y & -\frac{17}{14} \approx -1.2 & 0 \end{array}$ and $\begin{array}{c|c|c} x & 0 & \frac{17}{2} = 8.5 \\ \hline y & 17 & 0 \end{array}$



3. Describe each of the following systems as either dependent, inconsistent, or independent and consistent. Show how you justify your conclusion.

a. $\begin{array}{l} y = -\frac{17}{5}x \\ 17x + 5y = 0 \end{array}$

SOLN: Dependent: same slope (-17/5) and some y-intercept (0,0).

b. $\begin{array}{l} y = -\frac{17}{5}x \\ 5x + 17y = 0 \end{array}$

SOLN: Independent and consistent (not parallel.)

c. $\begin{array}{l} y = -\frac{17}{5}x + 1 \\ 17x + 5y = 1 \end{array}$

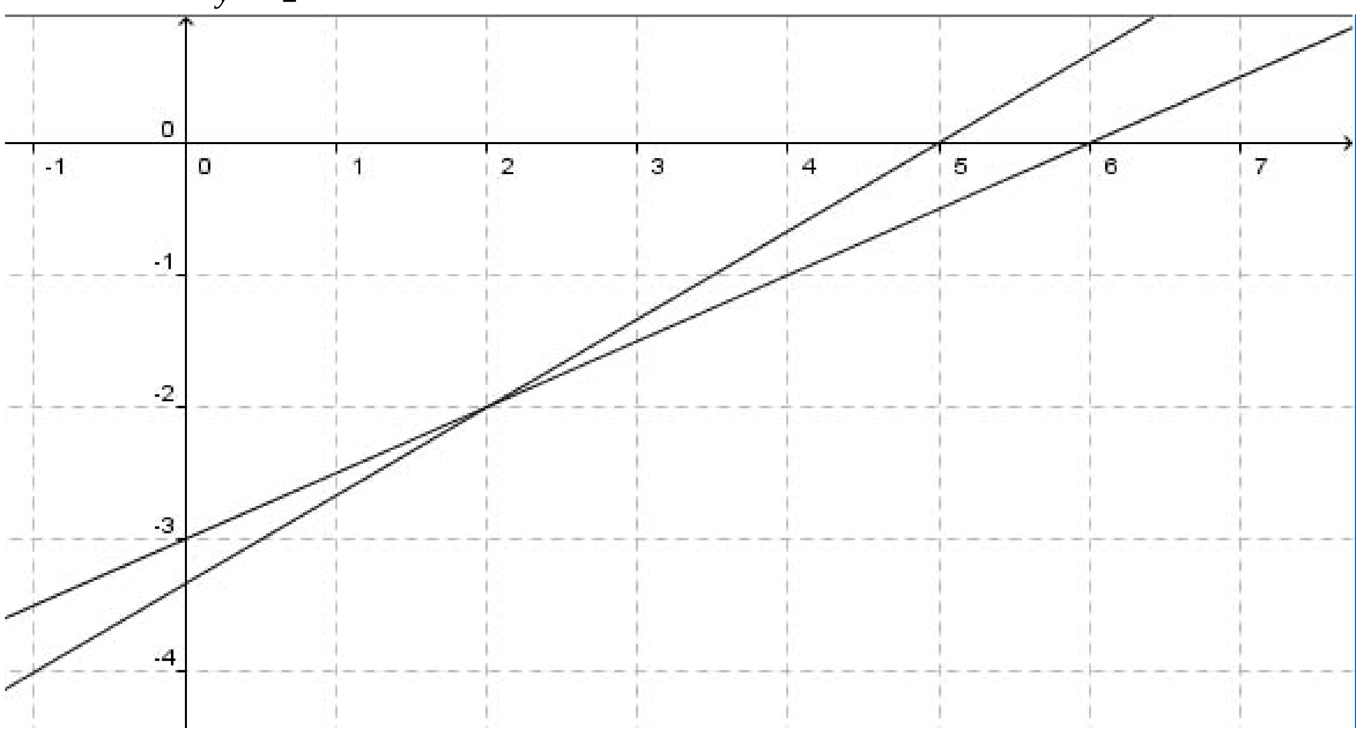
SOLN: Inconsistent. Parallel with different y-intercepts.

d. $\begin{array}{l} y = \frac{17}{5}x \\ 5x + 17y = 1 \end{array}$

SOLN: Independent and consistent. (not parallel)

4. Solve the system of equations by elimination, then illustrate in a graph how the two lines intersect at the solution point. Give coordinates of the solution: $\begin{array}{r} 2x - 3y = 10 \\ x - 2y = 6 \end{array}$

SOLN: $\begin{array}{r} 2x - 3y = 10 \\ -2x + 4y = -12 \\ y = -2 \end{array}$ and so $2x + 6 = 10$ and so $x = 2$. Thus the intersection is at $(2, -2)$.



5. A concession stand sells apple juice and tangerine juice. The apple juice costs \$1.50 per cup and the tangerine juice costs \$0.85. If the concession stand made \$88.80 of revenue selling 80 apple juice and tangerine juice cups during a day, how many tangerine juice cups were sold?
SOLN: Let A represent the number of apple juice cups sold and let T represent the number of tangerine cups sold. Then $150A + 85T = 8880$ and $A + T = 80$. Substituting $A = 80 - T$ into the first equation we have $150(80 - T) + 85T = 12000 - 65T = 8880$ and thus $T = 3120/65 = 48$ so that $A = 80 - 48 = 32$.

6. Let N = number of nickels, D = number of dimes and Q = number of quarters. Then

$$\begin{array}{lll} N + D + Q = 24 & 2N + Q = 25 & \Rightarrow 10N + 5Q = 125 \Rightarrow 7N = 63 \\ 5N + 10D + 25Q = 300 & \Rightarrow 15N + 25Q = 310 & -3N - 5Q = -62 \\ \underline{N - D = 1} & & \end{array}$$

Thus $\underline{N = 9, \quad D = 8 \text{ and } Q = 7}$

7. $\begin{aligned} 3x-4y+5z&=17 \\ 7y-4z&=16 \\ 3z&=9 \end{aligned} \Rightarrow 7y-4(3)=16 \Rightarrow 3x-4(4)+5(3)=17$. So the sol'n is $\begin{aligned} x&=6 \\ y&=4 \\ z&=3 \end{aligned}$

8. $\left[\begin{array}{ccc|c} 2 & 4 & 8 & 8 \\ 3 & 0 & 0 & -12 \\ 0 & 5 & 7 & 17 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 4 & 8 & 8 \\ 0 & 12 & 24 & 48 \\ 0 & 5 & 7 & 17 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 4 & 8 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 5 & 7 & 17 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 4 & 8 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 3 & 3 \end{array}\right]$

So $\boxed{z=1}$, $\boxed{y=2}$ and $\boxed{x=-4}$

9. Solve the dependent system and express *x* and *y* in terms of *z*.

$$\begin{aligned} 2x-y+z&=10 \\ x+2y+2z&=5 \\ 8x+y+7z&=40 \end{aligned}$$

SOLN: $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 10 \\ 1 & 2 & 2 & 5 \\ 8 & 1 & 7 & 40 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -1 & 1 & 10 \\ 0 & 5 & 3 & 0 \\ 0 & 5 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -1 & 1 & 10 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ so that all solutions can be characterized as elements of the set $\{(x,y,z) \mid y=-3z/5, x=2z/5\}$

10. Graph the system of inequalities, label the vertices with coordinate values and shade the solution

$$\begin{aligned} 3y-4x&\le 9 \\ y+x&\le 10 \\ x\ge 0,\ \ y&\ge 0 \end{aligned}$$

region: