Math 40 - Chapter 1 Test – Spring '10 Name: ______________________________

Directions:
Show all work for full credit.
Write all responses on separate paper, with the exception of the line you draw for #10.
Do not use a calculator.
Use plenty of paper. Make large graphs. Don't crowd into a corner.

1. A bicycle messenger is paid \$100 per day plus \$15.33 for every hour she bicycles.
   a. Make a table of values tabulating her earnings *E* (in \$) in relation to the time *t* (in hours) that she bicycles during a day. Include values for $t = 0$, $t = 1$, $t = 4$ and $t = 8$.
   b. Write an equation for the relation between *E* and *t*.

2. Dan chooses to gamble \$100 on coin flips. The agreement is that he will win \$10 if the coin flip is "heads" and lose \$2.5 if the coin flip is "tails." To Dan's dismay, every flip of the coin is "tails."
   a. Write a formula for the gambling money, *M*, Dan has after flipping the coin *n* times.
   b. After how many flips will Dan go broke (lose all his gambling money) ?

3. Determine whether or not the table represents x and y in a linear relation. If it is linear, what is the constant slope? If it is not linear, how does the slope change?

x	0	1	10	40
y	100	97.8	78	12

4. The World Conservation Union estimates the current polar bear population of 22,000 bears will decline steadily by 40% over the next 30 years. Using these assumptions, give a linear equation modeling the polar bear population *P* in terms of the number of years from now, *t*.

5. The height above sea level *H*, in meters, of a rocket at time *t*, in seconds, is modeled by the linear equation, $H = 1000 + 1.5t$. At what time will the rocket be 10000 meters above sea level?

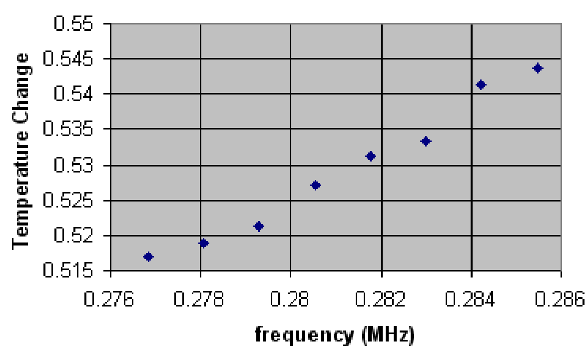
6. Graph the triangle with coordinates *A*(1,6), *B*(5,1) and *C*(-4,2) in the *xy*-plane. Is this a right triangle? Support your answer by computing slopes to show the triangle has a right angle or not.

7. Find the *x* coordinate at the point where $y = 3$ on a line through (11,5) with slope $\frac{3}{7}$.

8. Find the slope intercept form for the equation of the line through $\left(\frac{3}{4}, \frac{1}{2}\right)$ & $\left(\frac{1}{4}, \frac{7}{2}\right)$. Simplify.

9. A circle is centered at (6,0) and passes through (10,-3). Find an equation for the line tangent to the circle at (10,-3). It may help to make a graph.

10. The scatterplot shown at right plots experimental data for the temperature change (°C) in a substance hit by radiation of various frequencies (megahertz).
   a. Use a straight edge to estimate a line of best fit for the scatterplot.
   b. Use your line to predict the temperature change from a frequency of 0.284 MHz.
   c. Estimate the equation for the regression line.
   d. Use your equation to estimate the temperature change from a 0.28 MHz radiation



11. The data shown at right tabulates the second test scores T_1 and T_2 of 7 students on the first two tests in a college algebra.

a. Make a scatterplot showing the first test score along the horizontal axis and the second score along the vertical axis.

b. Draw a straight line approximating these data.

c. Calculate the equation for the line of best fit.

T_1	T_2
80	78
82	78
94	90
65	78
75	55
55	68
40	45

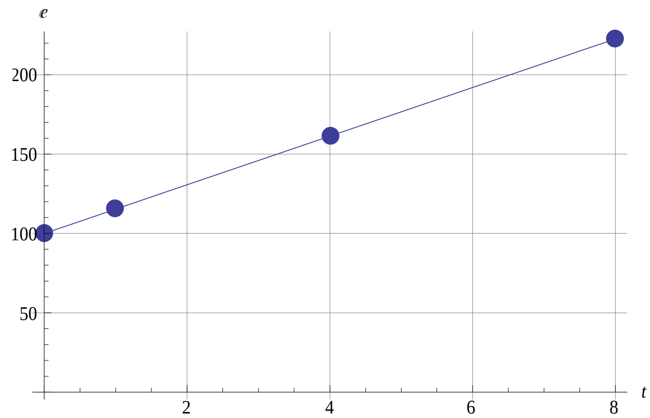
Math 40 - Chapter 1 Test Solutions – Spring 2010

1. A bicycle messenger is paid \$100 per day plus \$15.33 for every hour she bicycles.
   a. Make a table of values tabulating her earnings E (in \$) in relation to the time t (in hours) that she bicycles during a day. Include values for $t = 0$, $t = 1$, $t = 4$ and $t = 8$.

   SOLN:

t	0	1	4	8
E	100	115.33	161.32	222.64

   A graph wasn't asked for, but here one is anyway:



   b. Write an equation for the relation between E and t.
   $E = 15.33t + 100$

2. Dan chooses to gamble \$100 on coin flips. The agreement is that he will win \$10 if the coin flip is "heads" and lose \$2.5 if the coin flip is "tails." To Dan's dismay, every flip of the coin is "tails."
   a. Write a formula for the gambling money, M, Dan has after flipping the coin n times.
   SOLN: $M = 100 - 2.5n$
   b. After how many flips will Dan go broke (lose all his gambling money) ?
   $M = 0 \Leftrightarrow 100 - 2.5n = 0 \Leftrightarrow 2.5n = 100 \Leftrightarrow n = \dfrac{100}{2.5} = \dfrac{1000}{25} = 40$ coin flips before Dan is broke.

3. Determine whether or not the table represents x and y in a linear relation. If it is linear, what is the constant slope? If it is not linear, how does the slope change?

x	0	1	10	40
y	100	97.8	78	12

   SOLN: To be linear it is necessary and sufficient that the slopes of the line segments connecting the first point with each of the next three are equal.
   $\dfrac{97.8 - 100}{1 - 0} = -2.2 = \dfrac{78 - 100}{10 - 0} = -2.2 = \dfrac{12 - 100}{40 - 0} = -2.2$
   So, yes the table represents x and y in a linear relation.

4. The World Conservation Union estimates the current polar bear population of 22,000 bears will decline steadily by 40% over the next 30 years. Using these assumptions, give a linear equation modeling the polar bear population P in terms of the number of years from now, t.
   SOLN: 40% of 22,000 is 8800 bears, so we have a decrease of 8800 bears over 30 years giving a rate of change equal to $\dfrac{-880}{3} \approx -290$ bears/year. Thus we can model the bear population in t years by $P = 22000 - 293t$, where the "=" sign is understood to be approximate because the equation is just a model for some real world phenomenon.

5. The height above sea level H, in meters, of a rocket at time t, in seconds, is modeled by the linear equation, $H = 1000 + 1.5t$. At what time will the rocket be 10000 meters above sea level?

   SOLN: $H = 10000 \Leftrightarrow 1000 + 1.5t = 10000 \Leftrightarrow 1.5t = 9000 \Leftrightarrow t = \dfrac{9000}{1.5} = \dfrac{18000}{3} = 6000$ sec.

6. Graph the triangle with coordinates $A(1,6)$, $B(5,1)$ and $C(-4,2)$ in the xy-plane. Is this a right triangle? Support your answer by computing slopes to show the triangle has a right angle or not.

   SOLN $m(AB) = \dfrac{1-6}{5-1} = -\dfrac{5}{4}$ and $m(AC) = \dfrac{6-2}{1-(-4)} = \dfrac{4}{5}$ are negative reciprocals, so AB and AC are perpendicular – they meet in a right angle - so this is a right triangle.

7. Find the x coordinate at the point where $y = 3$ on a line through (11,5) with slope $\dfrac{3}{7}$.

   SOLN: An equation for the line can be found by using the point-slope formula:

   $y - y_1 = m(x - x_1) \Leftrightarrow y - 5 = \dfrac{3}{7}(x - 11)$. Plug in $y = 3$ and we have an equation we can solve for the x-coordinate: $3 - 5 = \dfrac{3}{7}(x-11) \Leftrightarrow x - 11 = \dfrac{-14}{3} \Leftrightarrow x = 11 - \dfrac{14}{3} = \boxed{\dfrac{19}{3} = 6\dfrac{1}{3} = 6.\overline{3}}$

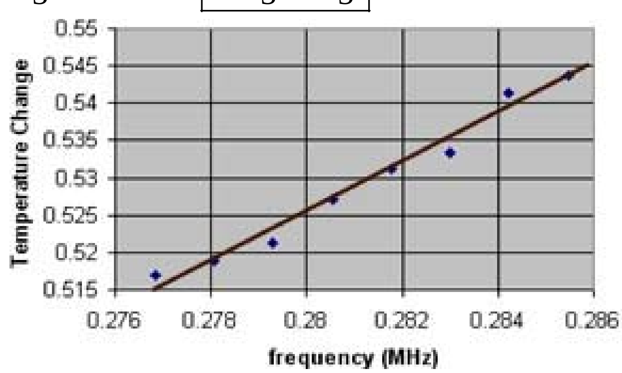
8. Find the slope intercept form for the equation of the line through $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$ & $\left(\dfrac{1}{4}, \dfrac{7}{2}\right)$. Simplify.

$$m = \frac{\frac{7}{2} - \frac{1}{2}}{\frac{1}{4} - \frac{3}{4}} = \frac{6/2}{-2/4} = -\frac{3}{1/2} = -\frac{3}{1}\cdot\frac{2}{1} = -6 \text{ so } y - \frac{1}{2} = -6\left(x - \frac{3}{4}\right) \Leftrightarrow y = -6x + \frac{9}{2} + \frac{1}{2} \Leftrightarrow \boxed{y = -6x + 5}$$

9. A circle is centered at (6,0) and passes through (10,-3). Find an equation for the line tangent to the circle at (10,-3). It may help to make a graph.

   SOLN: The slope of the radius line is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{-3-0}{10-6} = \dfrac{-3}{4}$ so the perpendicular slope is $m_\perp = \dfrac{4}{3}$ and a point slope form for the equation is $y - (-3) = \dfrac{4}{3}(x - 10) \Leftrightarrow \boxed{y = \dfrac{4}{3}x - \dfrac{49}{3}}$ (slope-intercept)

10. The scatterplot shown at right plots experimental data for the temperature change (°C) in a substance hit by radiation of various frequencies (megahertz).



   a. Use a straight edge to estimate a line of best fit for the scatterplot.
      SOLN: Connecting the second and last dots works pretty well. You end up with a couple of points above, a couple below and a couple right on the line.
   b. Use your line to predict the temperature change from a frequency of 0.284 MHz.
      SOLN: About 0.540 °C temperature change.
   c. Estimate the equation for the regression line.

      SOLN: Taking the slope to be $\dfrac{0.544 - 0.519}{0.2855 - 0.2780} = \dfrac{0.025}{0.0075} = \dfrac{250}{75} = \dfrac{10}{3} \approx 3.3^\circ$ C/MHz

      and plugging into the point slope formula: $T - 0.519 \approx 3.3(f - 0.278) \Leftrightarrow T = 3.3f - 0.40$
   d. Use your equation to estimate the temperature change from a 0.28 MHz radiation
      SOLN: $T \approx 3.3(0.28) - 0.40 \approx 0.92 - 0.4 = 0.52^\circ$C