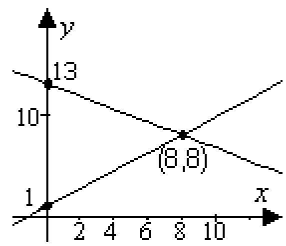
Math 40 Final Exam – Spring '10 Name: ________________________________

Directions: Write all responses on separate paper. Show work for credit.

1. Consider the line in *xy*-plane which is the solution set for the equation to $\frac{x}{17}+\frac{y}{51}=1$.
   a. What are the coordinates where the line crosses the coordinate axes? That is, what are the coordinates of the *x*-intercept and the *y*-intercept?
   b. What is the slope of the line?
   c. Write an equation for the line in slope-intercept form.
   d. Write an equation for the line perpendicular to this line and passing through (0,0).

2. The graph to the right shows the solution to a system of equations. Write the system in standard form, that is, find values of A, B, C, D, E, F so that the system is in the form:



$$Ax + By = C$$
$$Dx + Ey = F$$

3. Solve the system by back substitution:
$$\begin{aligned} 3x-4y+5z&=30\\ 7y-4z&=6\\ 2z&=18 \end{aligned}$$

4. Solve the system using either elimination, substitution, or row reduction on an augmented matrix.
$$\begin{aligned} 2x-y+3z&=38\\ x+3y+2z&=52\\ 4x-2y-z&=-43 \end{aligned}$$

5. Consider the parabola in *xy*-plane which is the solution set for the equation to $y=-x^2-2x+15$.
   a. Find the coordinates of the y–intercept and the *x*–intercepts for the parabola.
   b. Find the coordinates of the vertex for this parabola.
   c. Sketch a graph showing these features.

6. Consider the parabola in *xy*-plane which is the solution set for the equation to $y=4x^2-16x-33$.
   a. Find the coordinates of the y–intercept and the *x*–intercepts for the parabola.
   b. Find the coordinates of the vertex for this parabola.
   c. Sketch a graph showing these features.

7. Consider the parabola passing through the points (0, –14), (1, –4) and (4, 2).
   a. Set up a system of linear equations in *a*, *b*, and *c* to find an equation of the form $y = ax^2 + bx + c$ for the parabola.
   b. What are the coordinates of the vertex for this parabola?

8. Consider the quadratic equation $\left(x-\frac{3}{4}\right)^2=\frac{3}{8}$.
   a. Solve the equation and write the solutions in simplest radical form.
   b. Solve the inequality, $\left(x-\frac{3}{4}\right)^2\le\frac{3}{8}$, and write the solutions using interval notation.

9. Write a quadratic inequality whose solution is $x \in [-1, 7]$

10. Consider the function $f(x) = 4 - \sqrt{25 - x^2}$.

    a. Evaluate $f(-5)$, $f(-4)$, $f(-3)$, $f(0)$, $f(3)$ $f(4)$, and $f(5)$, and summarize your results in a table of x and y values.
    b. Plot these points in the *xy*-coordinate plane and connect them with a smooth curve.
    c. What is the domain of the function? What is the range?

11. Consider the circle in the *xy*-plane with radius 5 and center at $(-2, 4)$.

    a. Use the standard form $(x-h)^2 + (y-k)^2 = r^2$ to write an equation for the circle
    b. Find the coordinates of the *x*-intercepts of the circle. That is, plug in $y = 0$ and solve for y.
    c. Find the coordinates of the *y*-intercepts of the circle.
    d. Construct a graph of the circle showing the coordinates of the center, intercepts and extreme left, right, top, and bottom points.

12. A population of alligator fish in a Mississippi estuary is modeled by the function, $P(t) = 117(1.023)^t$ where P is the population size and t is years since January 1, 2000.

    a. What was the population on January 1, 2000?
    b. What is the growth factor? What is the growth rate?
    c. What was the population after one month? (You'll need a calculator to compute this.)
    d. What will the population be on January 1, 2012?
    e. How long will it take the population to grow to 200?

13. Find values of a and b so that $g(x) = a(b)^x$ is an exponential function that fits this table of values:

x	0	10	20
$g(x)$	1.040	1.528	2.245

14. Solve each equation for x, do not approximate:

    a. $10^x = 120$
    b. $2^x = 1000$
    c. $\log_{10}(x) = 2$
    d. $\log_2(x - 3) = 8$

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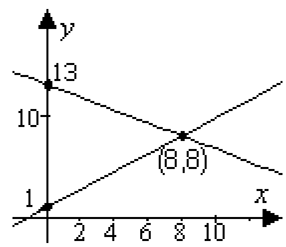
Directions: Write all responses on separate paper. Show work for credit.

1. Consider the line in *xy*-plane which is the solution set for the equation to $\frac{x}{17}+\frac{y}{51}=1$.
   a. What are the coordinates where the line crosses the coordinate axes? That is, what are the coordinates of the *x*-intercept and the *y*-intercept?
   SOLN: (17,0) and (0,51) are the *x*-intercept and the *y*-intercept, respectively.
   b. What is the slope of the line?
   SOLN: $m = -51/17 = -3$.
   c. Write an equation for the line in slope-intercept form.
   SOLN: $y = -3x + 51$.
   d. Write an equation for the line perpendicular to this line and passing through (0,0).
   SOLN: $y = x/3$.

2. The graph to the right shows the solution to a system of equations. Write the system in standard form, that is, find values of A, B, C, D, E, F so that the system is in the form:



$$Ax + By = C$$
$$Dx + Ey = F$$

SOLN: The slopes of the lines are $m_1 = 7/8$ and $m_2 = -5/8$ so the equations are

$y = \frac{7}{8}x + 1 \Leftrightarrow -7x + 8y = 8$ and $y = \frac{-5}{8}x + 13 \Leftrightarrow 5x + 8y = 104$ so the system can be written as

$$\boxed{\begin{aligned} -7x + 8y &= 8 \\ 5x + 8y &= 104 \end{aligned}}$$

3. Solve the system by back substitution:
$$\begin{aligned} 3x - 4y + 5z &= 30 \\ 7y - 4z &= 6 \\ 2z &= 18 \end{aligned}$$
SOLN: $z = 9$, $y = 6$ and $x = (30 - 45 + 24)/3 = 3$.

4. Solve the system using either elimination, substitution, or row reduction on an augmented matrix.
$$\begin{aligned} 2x - y + 3z &= 38 \\ x + 3y + 2z &= 52 \\ 4x - 2y - z &= -43 \end{aligned}$$
SOLN: $\begin{aligned} 2x - y + 3z &= 38 \\ x + 3y + 2z &= 52 \\ 4x - 2y - z &= -43 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 2 & -1 & 3 & 38 \\ 1 & 3 & 2 & 52 \\ 4 & -2 & -1 & -43 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 52 \\ 0 & 7 & 1 & 66 \\ 0 & 14 & 9 & 251 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 52 \\ 0 & 7 & 1 & 66 \\ 0 & 0 & -7 & -119 \end{array}\right]$

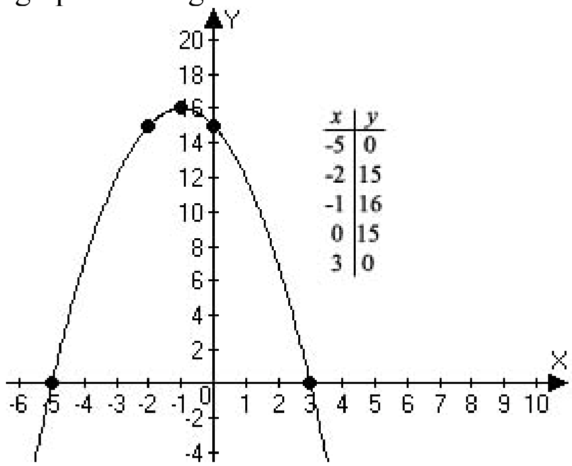
$\Rightarrow \boxed{z = 17,\ y = 7 \text{ and } x = -3}$.

5. Consider the parabola in *xy*-plane which is the solution set for the equation to $y = -x^2 - 2x + 15$.
   a. Find the coordinates of the *y*–intercept and the *x*–intercepts for the parabola.
   SOLN: The *y*–intercept is (0,15). $y = -x^2 - 2x + 15 = 16 - (x+1)^2$ is a difference of squares so $y = 16 - (x+1)^2 = (4 - (x+1))(4 + (x+1)) = (3 - x)(5 + x) = -(x+5)(x-3)$ and by the zero product principle, the *x*-intercepts are (–5,0) and (3,0).

b. Find the coordinates of the vertex for this parabola.
SOLN: $y = -x^2 - 2x + 15 = 16 - (x+1)^2$, so the vertex is (–1,16).

c. Sketch a graph showing these features.

SOLN:



6. Consider the parabola in *xy*-plane which is the solution set for the equation to $y = 4x^2 - 16x - 33$.

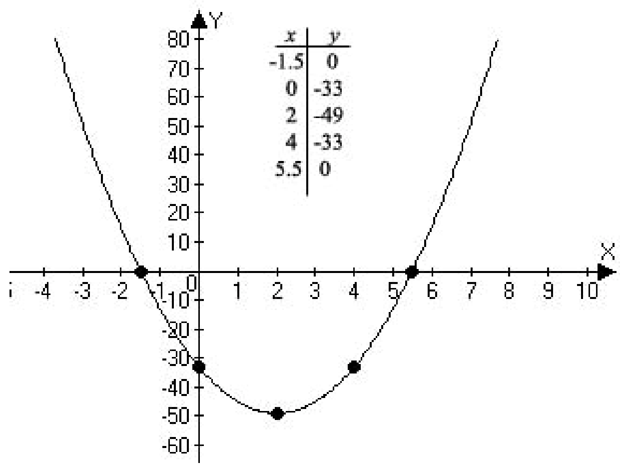
a. Find the coordinates of the *y*–intercept and the *x*–intercepts for the parabola.
SOLN: *y*-intercept is (0,–33). To find *x*-intercepts, solve

$$y = 4x^2 - 16x - 33 = 0 \Leftrightarrow (x-2)^2 = \frac{33}{4} + 4 \Leftrightarrow x = 2 \pm \sqrt{\frac{49}{4}} = \frac{11}{2} \text{ or } -\frac{3}{2}$$

b. Find the coordinates of the vertex for this parabola.
SOLN: $y = 4x^2 - 16x - 33 = 4(x-2)^2 - 49$ has vertex (2,–49).

c. Sketch a graph showing these features.

SOLN:



7. Consider the parabola passing through the points (0, –14), (1, –4) and (4, 2).

a. Set up a system of linear equations in *a*, *b*, and *c* to find an equation of the form $y = ax^2 + bx + c$ for the parabola.

SOLN: $\begin{array}{r} c = -14 \\ a + b + c = -4 \\ 16a + 4b + c = 2 \end{array}$ or $\begin{array}{r} a + b = 10 \\ 16a + 4b = 16 \end{array}$ or $\begin{array}{r} a + b = 10 \\ 4a + b = 4 \end{array}$ So $a = -2$, $b = 12$ and $c = -14$.

b. What are the coordinates of the vertex for this parabola?
SOLN $y = -2x^2 + 12x - 14 = -2(x-3)^2 + 4$ so the vertex is where $h = 3$ and $k = 4$, at (3,4).

8. Consider the quadratic equation $\left(x-\frac{3}{4}\right)^2=\frac{3}{8}$.

   a. Solve the equation and write the solutions in simplest radical form.

   SOLN: $\left(x-\frac{3}{4}\right)^2=\frac{3}{8} \Leftrightarrow x-\frac{3}{4}=\pm\frac{\sqrt{3}}{2\sqrt{2}} \Leftrightarrow \boxed{x=\frac{3}{4}\pm\frac{\sqrt{6}}{4}}$

   b. Solve the inequality, $\left(x-\frac{3}{4}\right)^2 \le \frac{3}{8}$, and write the solutions using interval notation.

   SOLN: $\left[\frac{3}{4}-\frac{\sqrt{6}}{4}, \frac{3}{4}+\frac{\sqrt{6}}{4}\right]$

9. Write a quadratic inequality whose solution is $x \in [-1,7]$

   SOLN: $(x + 1)(x - 7) \le 0$.

10. Consider the function $f(x)=4-\sqrt{25-x^2}$.

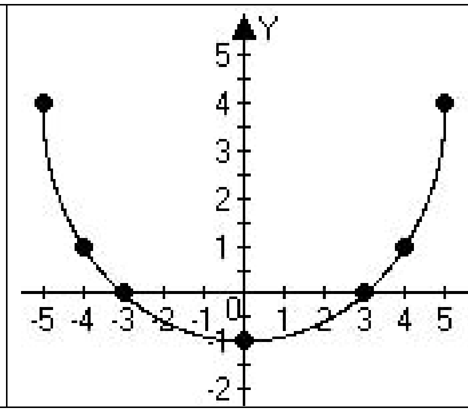
   a. Evaluate $f(-5)$, $f(-4)$, $f(-3)$, $f(0)$, $f(3)$ $f(4)$, and $f(5)$, tabulate your results.

   $f(\pm 5)=4-\sqrt{25-25}=4$, $f(\pm 4)=4-\sqrt{25-16}=1$, $f(\pm 3)=4-\sqrt{25-9}=0$ and $f(0)=-1$

x	-5	-4	-3	0	3	4	5
$f(x)$	4	1	0	-1	0	1	4

   b. Plot these points in the *xy*-coordinate plane and connect them with a smooth curve.

   c. What is the domain of the function? What is the range?
   SOLN: The domain is [–5,5] and the range is [–1,4]



11. Consider the circle in the *xy*-plane with radius 5 and center at $(-2,4)$.

   a. Use the standard form $(x-h)^2+(y-k)^2=r^2$ to write an equation for the circle.
   SOLN: $(x+2)^2+(y-4)^2=25$

   b. Find the coordinates of the *x*-intercepts.

   SOLN: $(x+2)^2+(0-4)^2=25$
   $\Leftrightarrow (x+2)^2=9 \Leftrightarrow x=-2\pm 3$

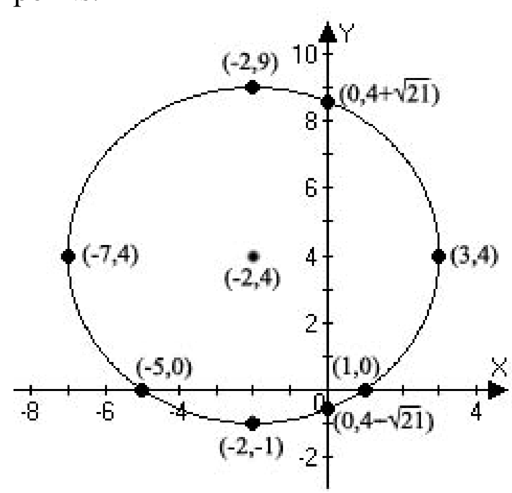
   so the *x*-intercepts are $(-5,0)$ and $(1,0)$

   c. Find the coordinates of the *y*-intercepts of the circle. SOLN: These are $\left(0,4\pm\sqrt{21}\right)$

   $(0+2)^2+(y-4)^2=25 \Leftrightarrow$

   $(y-4)^2=21 \Leftrightarrow \boxed{y=4\pm\sqrt{21}}$

   d. Construct a graph of the circle showing the coordinates of the center, intercepts and extreme left, right, top, and bottom points.



12. A population of alligator fish in a Mississippi estuary is modeled by the function, $P(t) = 117(1.023)^t$ where P is the population size and t is years since January 1, 2000.

a. What was the population on January 1, 2000?
SOLN: $P(0) = 117$

b. What is the growth factor? What is the growth rate?
SOLN: The growth factor is 1.023 and the growth rate is 2.3%

c. What was the population after one month? (You'll need a calculator to compute this.)
SOLN: $P\left(\frac{1}{12}\right) = 117(1.023)^{1/12} \approx 117(1.0009945418) \approx 117$, so no growth the first month

d. What will the population be on January 1, 2012?
SOLN $P(12) = 117(1.023)^{12} \approx 117(1.3137345) \approx 154$

e. How long will it take the population to grow to 200?
SOLN:
$$P(t) = 117(1.023)^t = 200 \Leftrightarrow (1.023)^t = \frac{200}{117} \Leftrightarrow t = \log_{1.023}\left(\frac{200}{117}\right) = \frac{\log(200/117)}{\log 1.023} \approx 23.6 \text{ yrs.}$$

13. Find values of a and b so that $g(x) = a(b)^x$ is an exponential function that fits this table of values:

x	0	10	20
$g(x)$	1.040	1.528	2.245

SOLN: $g(0) = a(b)^0 = 1.040$ and $g(10) = 1.04(b)^{10} = 1.528$ means
$b^{10} = \frac{1.528}{1.04} \approx 1.46923 \Rightarrow b = (1.43923)^{1/10} \approx 1.0392$ so $g(x) = 1.04(1.0392)^x$

14. Solve each equation for x , do not approximate:

a. $10^x = 120$
SOLN: $x = \log_{10}(120)$

b. $2^x = 1000$
SOLN: $x = \log_2(1000)$

c. $\log_{10}(x) = 2$
SOLN: $x = 10^2 = 100$.

d. $\log_2(x - 3) = 8$
SOLN: $x = 2^8 + 3 = 259$