Math 40 – Chapter 8 and 10.2 Test – Spring '09 Name________________________
Show your work for credit. Write all responses on separate paper.

In 1-2, use the method of extracting roots to solve the equation.

1. $(x-1)^2 = 12$

2. $2-(5x-2)^2 = 11$

In 3-4, use the method of completing the square to solve the equation.

3. $x^2 + 20x = 0$

4. $4x^2 + 20x + 1 = 0$

5. Two legs of a right triangle are represented as $2x-1$ and $4x$ while the hypotenuse is $5x-2$. Find all possible values of x.

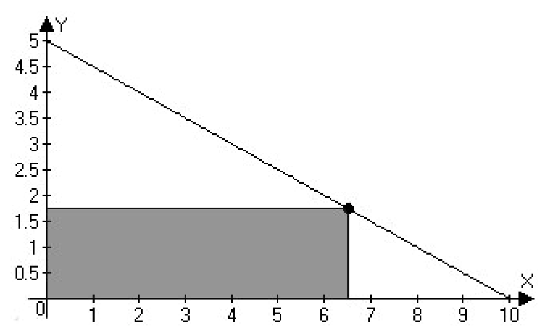
6. Let x represent the demand (the number the public will buy) for radios and let p represent the price for a radio. Suppose the price decreases linearly with the demand according to the formula $p = (-1/3)x + 40$. Then the revenue from selling x radishes is $R(x) = (-1/3)x^2 + 40x$.
   a. Find the values of $R(9)$ and $R(12)$.

   b. How many radios must be sold in order for the revenue to be $525?

   c. How many radios will be sold in order maximize the revenue?

   d. What is the maximum revenue?

7. A point from the first quadrant is selected on the line $y = -\frac{1}{2}x + 5$. Lines are drawn from this point parallel to the axes to form a rectangle under the line in the first quadrant. Among all such rectangles, find the length x and height y of the rectangle with the maximum area. What is that maximum area?



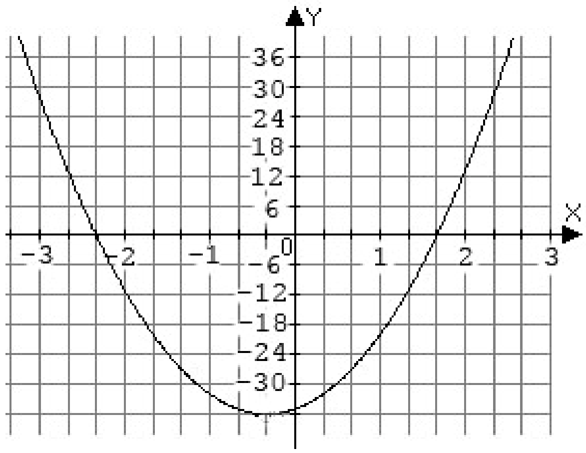
In problems 8-9, find all solutions to the equation.

8. $(2x+3)^2 + 7(2x+3) + 12 = 0$

9. $36x^4 - 37x^2 + 9 = 0$

For problems 10 and 11, write an equation for the parabola shown
   a. in vertex form: $y = a(x-h)^2 + k$
   b. in descending powers form: $y = ax^2 + bx + c$.
   c. in factored form: $y = a(x - r_1)(x - r_2)$

10.



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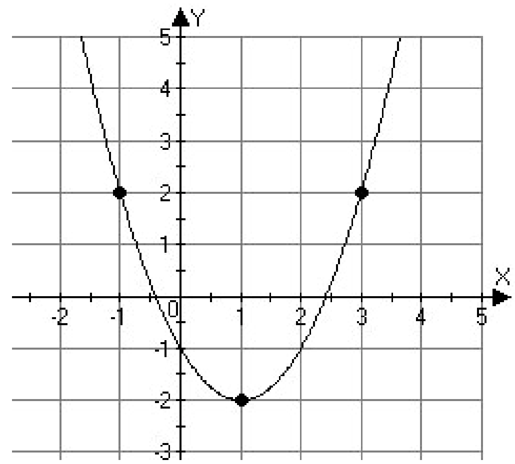


12. Sketch a graph of the parabola $f(x) = -2(4x-3)(2x+3)$ and use the graph to solve the inequality, $f(x) \le 0$ and write the solution using interval notation.

13. Sketch a graph of the parabola $f(x) = -x^2 + 2x - 1$ and use the graph to solve the inequality, $f(x) \le 0$ and write the solution using interval notation.

14. Find an equation for the circle centered at (8,15) with radius 17. Find four points on the circumference of the circle with integer coordinates.

15. Find the center and radius of the circle whose equation is $x^2 - 10x + y^2 - 24y = 0$ and write the equation in standard form: $(x-h)^2 + (y-k)^2 = r^2$

Math 40 – Chapter 8 and 10.2 Test Solutions.

In 1-2, use the method of extracting roots to solve the equation.

1. $(x-1)^2 = 12$

   SOLN: $x-1=\pm\sqrt{12} \Leftrightarrow \boxed{x=1\pm 2\sqrt{3}}$

2. $2-(5x-2)^2 = 11$

   SOLN: $(5x-2)^2 = -9 \Leftrightarrow 5x-2=\pm\sqrt{-9} \Leftrightarrow 5x = 2\pm\sqrt{9}\,i \Leftrightarrow \boxed{x=\frac{2}{5}\pm\frac{3}{5}i}$

In 3-4, use the method of completing the square to solve the equation.

3. $x^2+20x=0$

   SOLN: $x^2+20x+100=100 \Leftrightarrow (x+10)^2=100 \Leftrightarrow x+10=\pm 10 \Leftrightarrow x=-10\pm 10$

   so $x = 0$ or $x = -20$.

4. $4x^2+20x+1=0$

   SOLN: $x^2+5x+\frac{1}{4}=0 \Leftrightarrow x^2+5x+\left(\frac{5}{2}\right)^2 = -\frac{1}{4}+\frac{25}{4} \Leftrightarrow \left(x+\frac{5}{2}\right)^2=\frac{24}{4}$

   $\Leftrightarrow x+\frac{5}{2}=\pm\sqrt{6} \Leftrightarrow \boxed{x=-\frac{5}{2}\pm\sqrt{6}}$

5. Two legs of a right triangle are represented as $2x-1$ and $4x$ while the hypotenuse is $5x-2$. Find all possible values of x.

   SOLN: $(2x-1)^2+(4x)^2=(5x-2)^2 \Leftrightarrow 4x^2-4x+1+16x^2=25x^2-20x+4$

   $\Leftrightarrow 0=5x^2-16x+3 \Leftrightarrow (x-3)(5x-1)=0 \Leftarrow \boxed{x=3}$ The $x=\frac{1}{5}$ solution doesn't

   work since all sides must have positive length and $2/5 - 1 < 0$.

6. Let x represent the demand (the number the public will buy) for radios and let p represent the price for a radio. Suppose the price decreases linearly with the demand according to the formula $p = (-1/3)x + 40$. Then the revenue from selling x radishes is $R(x) = (-1/3)x^2 + 40x$.

   a. Find the values of $R(9)$ and $R(12)$.

      SOLN: $R(9) = (-1/3)9^2 + 40(9) = (-1/3)81 + 360 = -27 + 360 = \$333$

      $R(12) = (-1/3)12^2 + 40(12) = (-1/3)144 + 480 = -48 + 480 = \$432$

   b. How many radios must be sold in order for the revenue to be \$525?

      SOLN: $R(x) = 525$ means that $(-1/3)x^2 + 40x = 525$. Multiplying both sides of the equation by -3 yields $x^2 - 120x = -1575$. Completing the square leads to $x^2 - 120x + 3600 = 3600-1575$ or $(x - 60)^2 = 2025$ and since x can't be negative, $x = 60 + 45 = 105$ radios.

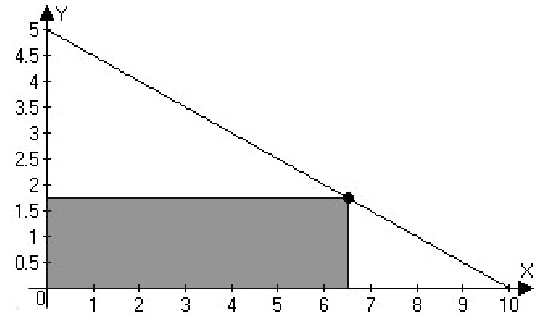
   c. How many radios will be sold in order maximize the revenue?

      SOLN: From the vertex form, $R(x) = (-1/3)(x - 60)^2 + 1200$ we see that if 60 radios are made then the revenue reaches a maximum value.

   d. What is the maximum revenue?

      SOLN \$1200.

7. A point from the first quadrant is selected on the line $y = -\frac{1}{2}x + 6$. Lines are drawn from this point parallel to the axes to form a rectangle under the line in the first quadrant. Among all such rectangles, find the length x and height y of the rectangle with the maximum area. What is that maximum area?



SOLN: The height of the rectangle is the y coordinate and the width is the x coordinate, so the area is width * height = xy

$$= x\left(-\frac{1}{2}x + 6\right) = -\frac{1}{2}x^2 + 6x = -\frac{1}{2}\left(x^2 - 12x\right) = -\frac{1}{2}\left(x^2 - 12x + 36\right) + \frac{1}{2}(36) = -\frac{1}{2}(x-6)^2 + 18$$

Now, in fact, the line in the graph is $y = -\frac{1}{2}x + 5$ (y intercept in the graph is is 5, not 6.)

In this case the area is width * height = xy

$$= x\left(-\frac{1}{2}x + 5\right) = -\frac{1}{2}x^2 + 5x = -\frac{1}{2}\left(x^2 - 10x\right) = -\frac{1}{2}\left(x^2 - 10x + 25\right) + \frac{1}{2}(25) = -\frac{1}{2}(x-5)^2 + \frac{25}{2}$$

So in the case of the equation the rectangle of maximum area is achieved with 3 * 6 = 18.

In the case of the graph the rectangle of maximum area is achieved with a $5 * \frac{5}{2} = \frac{25}{2}$

In problems 8-9, find all solutions to the equation.

8. $(2x+3)^2 + 7(2x+3) + 12 = 0$

SOLN: Let $y = 2x + 3$. Then the equation is $y^2 + 7y + 12 = 0$, or $(y + 3)(y + 4) = 0$ So by the zero product principle, either $y = -3$ or $y = -4$. Substituting $y = 2x + 3$ we have $y = -3$ or $y = -4$ become $2x + 3 = -3$ or $2x + 3 = -4$ which leads to the solutions $x = -3$ or $x = -7/2$.

9. $36x^4 - 37x^2 + 9 = 0$

SOLN: Note that the expression on the left side is quadratic in x^2. Also, 36*9=18*18=27*12 and 27+12=39 while 18+18=36, so it looks like the solutions are not rational. Completing the square, $36x^4 - 37x^2 + 9 = 0 \Leftrightarrow x^4 - \frac{37}{36}x^2 = -\frac{9}{36}$

$$x^4 - \frac{37}{36}x^2 + \left(\frac{37}{72}\right)^2 = \left(\frac{37}{72}\right)^2 - \frac{18}{72}\left(\frac{72}{72}\right) \Leftrightarrow \left(x^2 - \frac{37}{72}\right)^2 = \frac{1369}{5184} - \frac{1296}{5184}$$

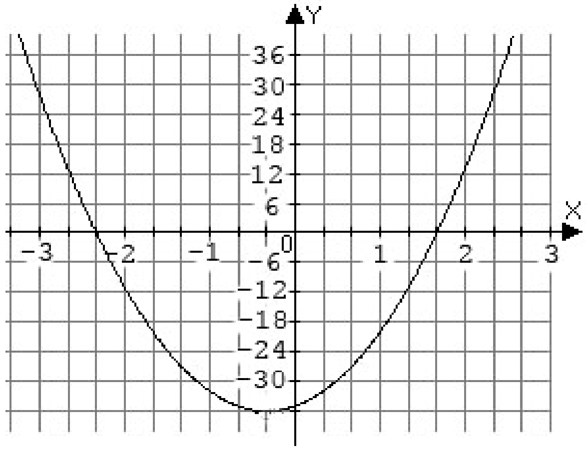
$$\Leftrightarrow x^2 - \frac{37}{72} = \pm\sqrt{\frac{73}{5184}} \Leftrightarrow x^2 = \frac{37}{72} \pm \frac{\sqrt{73}}{72} = \frac{74}{144} \pm \frac{2\sqrt{73}}{144} \Leftrightarrow \boxed{x = \frac{\pm\sqrt{74 \pm 2\sqrt{73}}}{12}}$$

For problems 10 and 11, write an equation for the parabola shown

a. in vertex form: $y = a(x - h)^2 + k$

b. in descending powers form: $y = ax^2 + bx + c$.

c. in factored form: $y = a(x - r_1)(x - r_2)$

10. 

(a) The vertex is at $\left(-\frac{1}{3}, -36\right)$ So the form is

$y = a\left(x + \frac{1}{3}\right)^2 - 36$ and it appears that there's a zero at $x = \frac{5}{3}$ so we substitute $(x, y) = \left(\frac{5}{3}, 0\right)$ and solve for *a*:

$0 = a\left(\frac{5}{3} + \frac{1}{3}\right)^2 - 36 \Leftrightarrow 4a = 36$

Thus the vertex form of the equation is

$y = 9\left(x + \frac{1}{3}\right)^2 - 36$

(b) Expanding the right hand side,

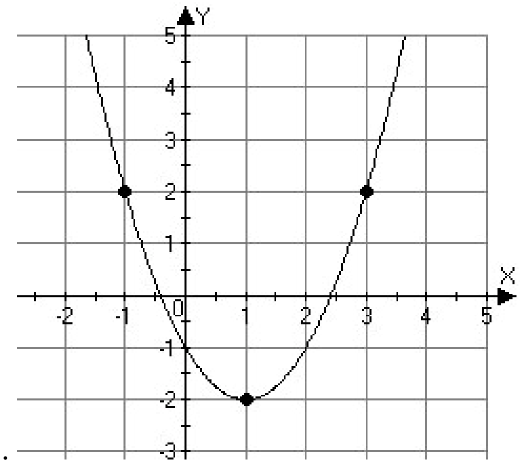
$y = 9\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) - 36$

$\boxed{y = 9x^2 + 6x - 35}$

(c) To find the zeros we solve $y = 0$, or

$9\left(x + \frac{1}{3}\right)^2 - 36 = 0 \Leftrightarrow x + \frac{1}{3} = \pm 2$

So $y = 9\left(x - \frac{5}{3}\right)\left(x + \frac{7}{3}\right) = (3x - 5)(3x + 7)$

11. 

a) The vertex is at $(1, -2)$ So the form is $y = a(x - 1)^2 - 2$ and it appears (3,2) are the coordinates of a point on the solutions curve, so we substitute $(x, y) = (3, 2)$ and solve for *a*:

$2 = a(3 - 1)^2 - 2 \Leftrightarrow 4a = 4 \Leftrightarrow a = 1$

Thus the vertex form of the equation is

$y = (x - 1)^2 - 2$

(b) Expanding the right hand side,

$y = x^2 - 2x + 1 - 2$

$\boxed{y = x^2 - 2x - 1}$

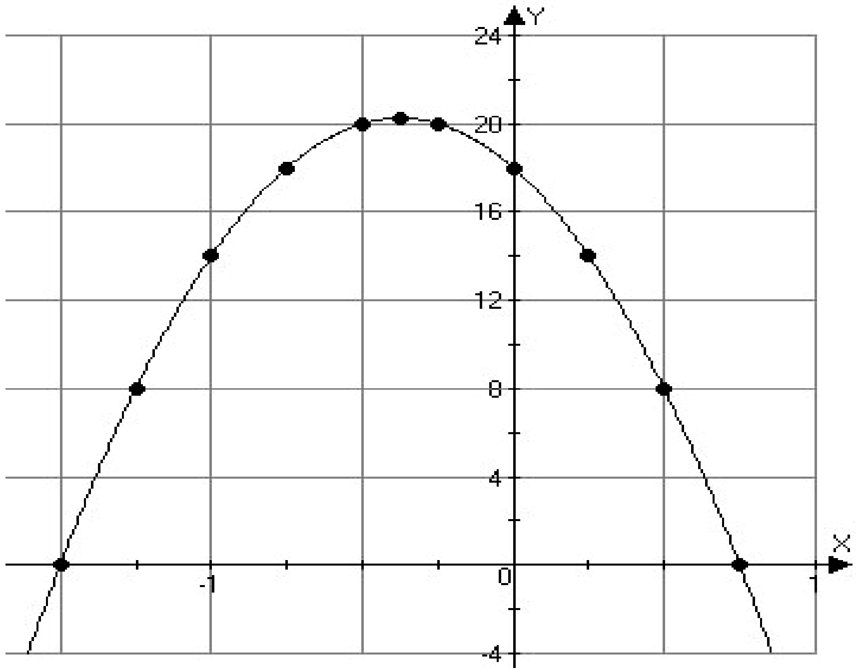
(c) To find the zeros we solve $y = 0$, or

$(x - 1)^2 - 2 = 0 \Leftrightarrow x = 1 \pm \sqrt{2}$

So $y = \left(x - 1 + \sqrt{2}\right)\left(x - 1 - \sqrt{2}\right)$

12. Sketch a graph of the parabola $f(x) = -2(4x-3)(2x+3)$ and use the graph to solve the inequality, $f(x) \leq 0$ and write the solution using interval notation.

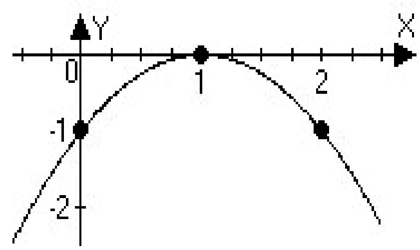
SOLN: Evidently, $y \leq 0$ if and only if $x \in \left(-\infty, \frac{3}{2}\right] \cup \left[\frac{3}{4}, \infty\right)$



x	7
−1.5	0
−1.25	8
−1	14
−0.75	18
−0.5	20
−0.375	20.25
−0.25	20
0	18
0.25	14
0.5	8
0.75	0

13. Sketch a graph of the parabola $f(x) = -x^2 + 2x - 1$ and use the graph to solve the inequality, $f(x) \leq 0$ and write the solution using interval notation.

SOLN: $f(x) = -x^2 + 2x - 1 = -(x-1)^2$ opens downward from a vertex at (1,0) so the inequality is true for all real number values of *x*.



14. Find an equation for the circle centered at (8,15) with radius 17. Find four points on the circumference of the circle with integer coordinates.

SOLN: $(x-8)^2 + (y-15)^2 = 17^2$ passes through (0,0), (0,30), (16,30) and (16,0).

15. Find the center and radius of the circle whose equation is $x^2 - 10x + y^2 - 24y = 0$ and write the equation in standard form: $(x-h)^2 + (y-k)^2 = r^2$

SOLN:
$$x^2 - 10x + 25 + y^2 - 24y + 144 = 0 + 25 + 144$$
$$(z-5)^2 + (y-12)^2 = 13^2$$

is the equation for a circle of radius 13 centered at (5, 12).