Math 40 – Final Exam – Spring '09 Name: _____
Directions: Show all work for credit. Write all responses on separate paper.

In problem 1 – 3, classify each as either a contradiction, an identity, or a conditional equation.

1. $\dfrac{x-1}{x-3}=2$

2. $\sqrt{(x-4)^2}=|x-4|$

3. $\dfrac{3x^2-8x+5}{x^2-3x+2}=\dfrac{4x-7}{x-2}$
   hint: multiplying both sides of an equation by zero doesn't yield an equivalent equation.

4. Given that a line has slope $-\dfrac{3}{8}$ and *y*-intercept $(0,9)$, complete the table:

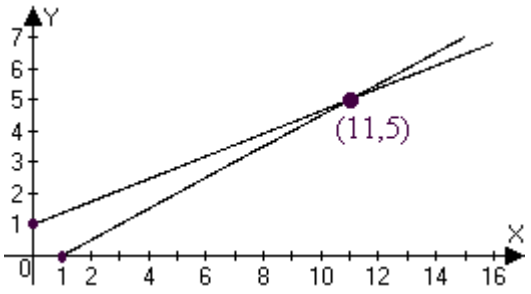
x	0	8	16	24
y				

5. Find an equation for the line passing through (1, 1) and parallel to $\dfrac{x}{12}+\dfrac{y}{144}=1$.

6. Find an equation for the line passing through (1, 1) and perpendicular to $\dfrac{x}{12}+\dfrac{y}{144}=1$.

7. Solve the system by graphing each equation and observing the coordinates of the point of intersection: $\begin{cases}2x+5y=25\\2x-5y=-5\end{cases}$

8. The graph at right shows two lines intersecting at (11,5).
   Write two equations: one for each line.



9. Solve the system by back substitution:
$$\begin{aligned}5x-7y+2z&=4\\8y-3z&=29\\4z&=36\end{aligned}$$

10. Solve the system using Gaussian elimination on an augmented matrix.

$$5x-3y+4z=24$$
$$7x-2y+z=48$$
$$x+4y-8z=39$$

11. A coin collection consists of pennies, nickels and dimes. There are 40 coins. The total value of the coins $2.53. The number of dimes is 1 more than twice the number of pennies. Set up and solve a system of equations to find how many of each type of coin there are.

12. Solve the absolute value equation: $\left|x-\frac{1}{2}\right|+\frac{1}{2}=\frac{3}{4}$.

Write the solution set using set notation.

13. Solve the absolute value inequality: $\left|x-\frac{1}{2}\right|+\frac{1}{2}<\frac{3}{4}$.

Write the solution using interval notation.

14. Solve the equation $\left(x+\frac{5}{4}\right)^2=\frac{1}{3}$.

15. Solve the inequality $6x-x^2<-1$. Write the solution using interval notation.

16. Evaluate and simplify $f\left(\frac{1}{9}\right)$, where $f(x)=\frac{5}{81}-\sqrt{\frac{10}{729}-x^2}$

17. Write an equation for the circle in the *x*-*y* plane of radius 8 and center at $(3,10)$.

18. Solve the equation using the method of completing the square: $2x^2-2x-1=0$

19. Find the vertex and *x*-intercepts of $y=-5(x-4)^2+80$ and graph the parabola.

20. What real numbers are not in the domain of $R(x)=\frac{x^2-2}{x^2-x-22}$?

SOLUTIONS

In problem 1 – 3, classify each as either a contradiction, an identity, or a conditional equation.

1. $\dfrac{x-1}{x-3}=2 \Leftrightarrow x-1=2(x-3) \Leftrightarrow x-1=2x-6 \Leftrightarrow x=5$ is conditional

2. $\sqrt{(x-4)^2}=|x-4|$ is an identity since, in general, $\sqrt{u^2}=|u|$

3. $\dfrac{3x^2-8x+5}{x^2-3x+2}=\dfrac{4x-7}{x-2} \Leftrightarrow \dfrac{(3x-5)(x-1)}{(x-1)(x-2)}=\dfrac{4x-7}{x-2} \Leftrightarrow \dfrac{3x-5}{x-2}=\dfrac{4x-7}{x-2}$ is a contradiction.

4. Given that a line has slope $-\dfrac{3}{8}$ and *y*-intercept $(0,9)$, complete the table:

x	0	8	16	24
y	**9**	**6**	**3**	**0**

5. Find an equation for the line passing through (1, 1) and parallel to $\dfrac{x}{12}+\dfrac{y}{144}=1$.

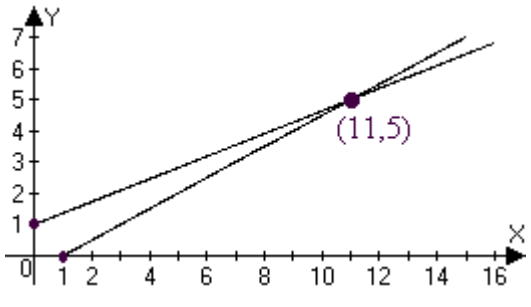
   SOLN: $\dfrac{x}{12}+\dfrac{y}{144}=1 \Leftrightarrow y=-12x+144$ has slope –12, so we plug into the point slope formula and get $\boxed{y-1=-12(x-1)} \Leftrightarrow \boxed{y=-12x+13}$

6. Find an equation for the line passing through (1, 1) and perpendicular to $\dfrac{x}{12}+\dfrac{y}{144}=1$.

   SOLN: $\dfrac{x}{12}+\dfrac{y}{144}=1 \Leftrightarrow y=-12x+144$ has slope –12, so the perpendicular slope is 1/12 and we plud into the point slope formula to get $\boxed{y-1=\dfrac{1}{12}(x-1)} \Leftrightarrow \boxed{y=\dfrac{1}{12}x+\dfrac{11}{12}}$

7. Solve the system by graphing each equation and observing the coordinates of the point of intersection: $\boxed{\begin{aligned}2x+5y&=25\\2x-5y&=-5\end{aligned}}$

8. The graph at right shows two lines intersecting at (11,5).
   Write two equations: one for each line.
   SOLN: $y=\dfrac{4}{11}x+1$ and $y=\dfrac{1}{2}(x-1)$



9. Solve the system by back substitution:
$$\begin{aligned} 5x - 7y + 2z &= 4 \\ 8y - 3z &= 29 \\ 4z &= 36 \end{aligned}$$
SOLN: $z = 9$ implies that $8y = 29 + 27$ so that $y = 7$ and finally, $5x = 4 + 49 - 18$ so that $x = 7$. The solution is thus $(x, y, z) = (7, 7, 9)$.

10. Solve the system using Gaussian elimination on an augmented matrix.
$$\begin{aligned} 5x - 3y + 4z &= 24 \\ 7x - 2y + z &= 48 \\ x + 4y - 8z &= 39 \end{aligned}$$
SOLN: $\left[\begin{array}{ccc|c} 5 & -3 & 4 & 24 \\ 7 & -2 & 1 & 48 \\ 1 & 4 & -8 & 39 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -8 & 39 \\ 7 & -2 & 1 & 48 \\ 5 & -3 & 4 & 24 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -8 & 39 \\ 0 & 30 & -57 & 225 \\ 0 & 23 & -44 & 171 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -8 & 39 \\ 0 & 10 & -19 & 75 \\ 0 & 23 & -44 & 171 \end{array}\right]$
$\sim \left[\begin{array}{ccc|c} 1 & 4 & -8 & 39 \\ 0 & 10 & -19 & 75 \\ 0 & 0 & 3 & 15 \end{array}\right]$ So $z = 5$, $10y = 75 + 45$ means that $y = 12$ and so $x = 39 - 48 + 40 = 31$
and so the solution is $(x, y, z) = (5, 12, 31)$.

11. A coin collection consists of pennies, nickels and dimes. There are 40 coins. The total value of the coins \$2.53. The number of dimes is 1 more than twice the number of pennies. Set up and solve a system of equations to find how many of each type of coin there are.
SOLN: Let P, N and D represent the numbers of pennies, nickels and dimes, respectively. Then $P + N + D = 40$, $P + 5N + 10D = 253$, and $D = 2P + 1$. Substituting, we have $3P + N = 39$ and $21P + 5N = 243$. Solving the first of these for N and substituting into the second yields $21P + 5(39 - 3P) = 243$ or $6P = 48$ so $P = 8$, $D = 17$ and $N = 15$. To be sure this is $8 + 75 + 170 = 253$ cents.

12. Solve the absolute value equation: $\left|x - \frac{1}{2}\right| + \frac{1}{2} = \frac{3}{4}$.
Write the solution set using set notation.
SOLN: $\left|x - \frac{1}{2}\right| = \frac{1}{4} \Leftrightarrow x - \frac{1}{2} = \frac{1}{4}$ or $x - \frac{1}{2} = -\frac{1}{4} \Leftrightarrow \boxed{x = \frac{3}{4} \text{ or } x = \frac{1}{4}}$

13. Solve the absolute value inequality: $\left|x - \frac{1}{2}\right| + \frac{1}{2} < \frac{3}{4}$.
Write the solution using interval notation.
SOLN: $\left|x - \frac{1}{2}\right| < \frac{1}{4} \Leftrightarrow -\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4} \Leftrightarrow \frac{1}{4} < x < \frac{3}{4} \Leftrightarrow x \in \left(\frac{1}{4}, \frac{3}{4}\right)$

14. Solve the equation $\left(x + \frac{5}{4}\right)^2 = \frac{1}{3}$.
SOLN: $x + \frac{5}{4} = \pm\sqrt{\frac{1}{3}} \Leftrightarrow \boxed{x = -\frac{5}{4} \pm \frac{\sqrt{3}}{3}}$

15. Solve the inequality $6x - x^2 < -1$. Write the solution using interval notation.
SOLN:

$6x - x^2 < -1 \Leftrightarrow x^2 - 6x > 1 \Leftrightarrow x^2 - 6x + 9 > 10 \Leftrightarrow (x-3)^2 > 10 \Leftrightarrow \sqrt{(x-3)^2} > \sqrt{10} \Leftrightarrow |x-3| > \sqrt{10}$

This is means $x \in \left(-\infty, 3-\sqrt{10}\right) \cup \left(3+\sqrt{10}, \infty\right)$

16. Evaluate and simplify $f\left(\frac{1}{9}\right)$, where $f(x) = \frac{5}{81} - \sqrt{\frac{10}{729} - x^2}$

SOLN: $f\left(\frac{1}{9}\right) = \frac{5}{81} - \sqrt{\frac{10}{729} - \left(\frac{1}{9}\right)^2} = \frac{5}{81} - \sqrt{\frac{10}{729} - \frac{1}{81}} = \frac{5}{81} - \sqrt{\frac{10}{729} - \frac{9}{729}} = \frac{5}{81} - \frac{1}{27} = \frac{2}{81}$

17. Write an equation for the circle in the *x*-*y* plane of radius 8 and center at $(3,10)$.

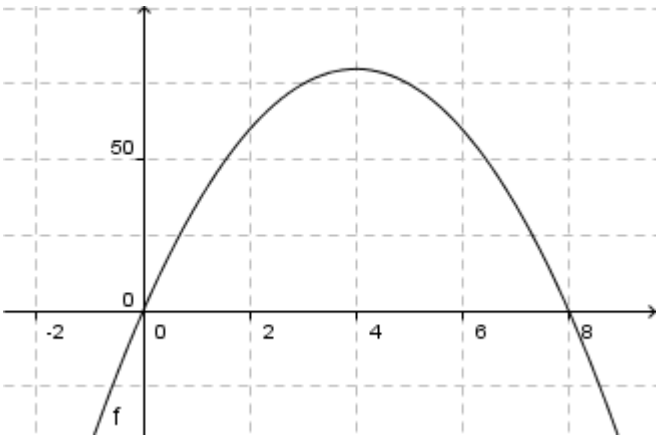
SOLN: $(x-3)^2 + (y-10)^2 = 64$

18. Solve the equation using the method of completing the square: $2x^2 - 2x - 1 = 0$

SOLN: $2x^2 - 2x - 1 = 0 \Leftrightarrow x^2 - x = \frac{1}{2} \Leftrightarrow x^2 - x + \frac{1}{4} = \frac{3}{4} \Leftrightarrow \left(x - \frac{1}{2}\right)^2 = \frac{3}{4} \Leftrightarrow \boxed{x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}}$

19. Find the vertex and *x*-intercepts of $y = -5(x-4)^2 + 80$ and graph the parabola.

SOLN: The vertex is (4,80) and the intercepts are (0,0) and (8,0).



20. What real numbers are not in the domain of $R(x) = \frac{x^2 - 2}{x^2 - x - 22}$?

SOLN: $x^2 - x - 22 = 0 \Leftrightarrow x^2 - x + \frac{1}{4} = 22 + \frac{1}{4} \Leftrightarrow \left(x - \frac{1}{2}\right)^2 = \frac{89}{4} \Leftrightarrow \boxed{x = \frac{1}{2} \pm \frac{\sqrt{89}}{2}}$

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