Math 40 **Name (Print):** ______________________

Exam 3: Chapter 3 Solutions

Write all responses on separate paper. Remember to organize your work clearly. You may *not* use your books, notes, or any calculator on this exam.

1. (21 points) Solve each equation for y by extracting roots:

   (a) $y^2 - 3 = 0$
   Solution:
   $y^2 = 3 \Leftrightarrow y = \pm\sqrt{3}$

   (b) $\left(y - \frac{1}{3}\right)^2 = \frac{1}{4}$

   $$y = \frac{1}{3} \pm \frac{1}{2} = \begin{cases} -\frac{1}{6} \\ \frac{5}{6} \end{cases}$$

   (c) $(y - x)^2 = 5$
   Solution:
   $y = x \pm \sqrt{5}$

2. (21 points) Decide whether to solve by factoring or completing the square and then solve.

   (a) $(v - 6)(v + 11) = -30$
   Solution: Start by writing the equation in standard form. This means expanding the product on the left side and combining like terms: $v^2 + 5v - 66 = -30$ and then adding 30 to both sides to get zero on one side: $v^2 + 5v - 36 = 0$. Now the middle term can be split up as $5v = 9v - 4v$ which makes a good substitution since $9(-4) = -36$. After splitting the middle term this way, the expression factors be grouping like so:

   $$\begin{aligned} v^2 + 5v - 36 &= 0 \\ v^2 + 9v - 4v - 36 &= 0 \\ v(v + 9) - 4(v + 9) &= 0 \\ (v - 4)(v + 9) &= 0 \end{aligned}$$

   Alternatively, one can complete the square, like so

   $$\begin{aligned} v^2 + 5v &= 36 \\ v^2 + 5v + \left(\frac{5}{2}\right)^2 &= 36 + \left(\frac{5}{2}\right)^2 \\ (v + \frac{5}{2})^2 &= 36 + \frac{25}{4} \\ v + \frac{5}{2} &= \pm\frac{13}{2} \\ v &= -\frac{5}{2} \pm \frac{13}{2} \end{aligned}$$

   Either way, the solutions are $v = 4$ or $v = -9$.

   (b) $x^2 + x - \frac{3}{4} = \frac{13}{36}$
   Solution: This one looks like a good candidate for completing the square. By adding 1 to bothe sides, we get $x^2 + x + \frac{1}{4} = \frac{49}{36} \Leftrightarrow \left(x + \frac{1}{2}\right)^2 = \frac{49}{36} \Leftrightarrow x + \frac{1}{2} = \pm\frac{7}{6} \Leftrightarrow x = \frac{-3 \pm 7}{6}$

Evidently, the solutions are rational numbers, so we could have solved by factoring. To do this, it may help to first clear out the fractions by multiplying both sides by 36:

$$\begin{aligned} 36x^2 + 36x - 27 &= 13 \\ 36x^2 + 36x - 40 &= 0 \\ 9x^2 + 9x - 10 &= 0 \\ 9x^2 + 15x - 6x - 10 &= 0 \\ 3x(3x+5) - 2(3x+5) &= 0 \\ (3x-2)(3x+5) &= 0 \end{aligned}$$

Either way, the solutions are $x = \frac{-5}{3}$ and $x = \frac{2}{3}$

(c) $z(6z + 30) = (z - 15)^2$

Solution: First, this needs to be put in standard form:

$$\begin{aligned} z(6z+30) &= (z-15)^2 \\ 6z^2 + 30z &= z^2 - 30z + 225 \\ 5z^2 + 60z - 225 &= 0 \\ z^2 + 12z - 45 &= 0 \end{aligned}$$

This is a good candidate for completing the square:

$$\begin{aligned} z^2 + 12z &= 45 \\ z^2 + 12z + 6^2 &= 45 + 36 \\ (z+6)^2 &= 85 \\ z + 6 &= \pm\sqrt{81} \\ z &= -6 \pm 9 \end{aligned}$$

This too could have been solved by factoring: $z^2 + 12z - 45 = (z + 15)(z - 3) = 0 \Leftrightarrow z = -15$ or $z = 3$

3. (29 points) Let $y = 225 - x^2$.

(a) Find the coordinates of the x-intercepts of the graph. **Solution:** $(-15, 0), (15, 0)$

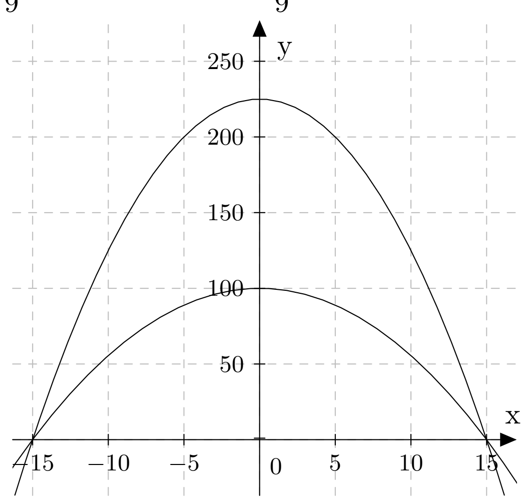
(b) Find the coordinates of the vertex of the graph. **Solution:** $(0, 225)$

(c) Make a table of at least five (x, y) solutions and use these to graph the parabola.

x	-15	-5	0	5	15
y	0	200	225	200	0

(d) Find the x-intercepts and vertex of $y = 100 - \frac{4}{9}x^2$ and sketch its graph together with the graph of the other parabola. What do you notice?

Solution: The x-intercepts are at $(-15, 0), (15, 0)$ and the vertex is at $(0, 100)$ It's a vertically compressed form of the the other parabola, since $\frac{4}{9}(225 - x^2) = 100 - \frac{4}{9}x^2$



4. (29 points) Let $y = x^2 - 5x$

   (a) Find the coordinates of the x-intercepts of the graph. **Solution:** $(0, 0), (5, 0)$

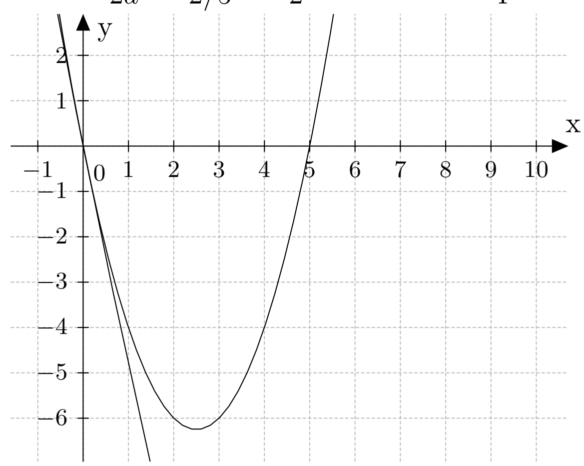
   (b) Find the coordinates of the vertex of the graph. **Solution:** The x-coordinate is halfway between the two intercepts, or you could use the formula, $x_v = -\dfrac{b}{2a} = \dfrac{5}{2}$. Plugging this into the equation yields $y_v = \dfrac{25}{4} - \dfrac{25}{2} = -\dfrac{25}{4}$ So the vertex is at $\left(\dfrac{5}{2}, -\dfrac{25}{4}\right)$.

   (c) Make a table of at least five (x, y) solutions and use these to graph the parabola.

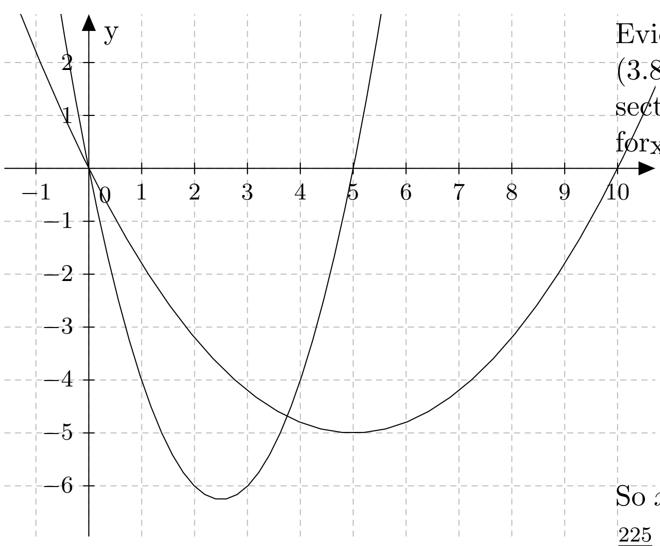
x	0	1	2	5/2	3	4	5
y	0	-4	-6	-6.25	-6	-4	0

   (d) Find the x-intercepts and vertex for $y = \frac{1}{5}x^2 - 5x$ and sketch its graph together with the graph of the other parabola. Use the graph to estimate the coordinates where the two parabolas intersect.

Solution: The intercepts are at $(0, 0), (25, 0)$ and the vertex is where $x_v = -\dfrac{b}{2a} = \dfrac{5}{2/5} = \dfrac{25}{2}$ and $y_v = -\dfrac{125}{4}$.



Problem 4d actually had a typo in it, which explains why it seems a bit odd. The second parabola's equation should have been $y = \frac{1}{5}x^2 - 2x$ so that its intercepts are at $(0, 0), (10, 0)$ and the vertex is at $(5, -5)$ and when the two parabolas are graphed together, we see this:



Evidently, the point of intersection is near $(3.8, -4.7)$. To find the exact point of intersection, set the y-coordinates equal and solve for x:

$$\begin{aligned} y &= y \\ x^2 - 5x &= \frac{1}{5}x^2 - 2x \\ 5x^2 - 25x &= x^2 - 10x \\ 4x^2 - 15x &= 0 \\ x(4x - 15) &= 0 \end{aligned}$$

So $x = \dfrac{15}{4} = 3.75$ making $y = \left(\frac{15}{4}\right)^2 - 5\left(\frac{15}{4}\right) = \frac{225}{16} - \frac{75}{4} = -\frac{75}{16} = -4.625$