Math 40 - Exam 2 Solutions

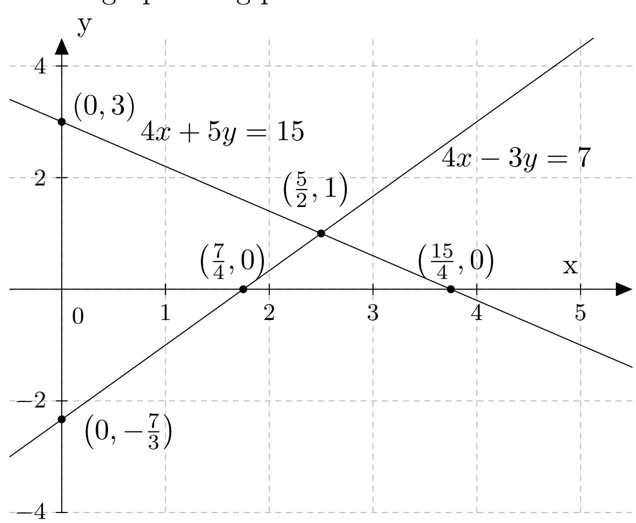
1. Consider the system of equations $\begin{aligned} 4x - 3y &= 7 \\ 4x + 5y &= 15 \end{aligned}$.

   (a) Make a table of values for each equation including at least 3 points in each.
   Solution:

$4x - 3y = 7$	x	0	$\frac{7}{4}$	$\frac{5}{2}$
	y	$-\frac{7}{3}$	0	1

$4x + 5y = 15$	x	0	$\frac{5}{2}$	$\frac{15}{4}$
	y	3	1	0

   (b) Construct a careful graph using points from the two tables to plot the lines.



   Solution:

   (c) Use the graph to estimate the solution to the system of equations.
   Solution: What luck! One of the points used in both tables is the same, and this is the point of intersection of the two lines: $\left(\frac{5}{2}, 1\right)$

2. A furniture store sends out 25 boxes containing either a chair or a table. The cost of sending a chair is \$8 and the cost of sending a table is \$12. The store spends \$280 to ship the 25 boxes. The steps below guide you to using the algebraic method to determine how many chairs were sent and how many tables were sent.

   (a) Introduce two variables to use in your system of equations. Name each variable and write a short description for what it represents.
   Solution: Let $x =$ the number of chairs sent, and $y =$ the number of tables sent

   (b) In terms of your variables, how much does it cost to send the chairs? The tables?
   Solution: The cost of sending the chairs is $\$8x$. The cost of sending the tables is $\$12y$.

   (c) Write a system of 2 linear equations in your 2 variables.
   Solution: $\begin{aligned} x + y &= 25 \\ 8x + 12y &= 280 \end{aligned}$.

   (d) Solve the system to determine how many chairs were sent and how many tables were sent.
   Solution: -8 times the first equation can be combined with the second equation by equating the sum of the left sides with the sum of the right sides, producing $4y = 80$ so $y = 20$ tables were sent, which means $x = 5$ chairs were sent.

3. Solve the system by elimination: $\begin{array}{r} 3x + 2y = -3 \\ 5x - 4y = 17 \end{array}$.

Solution: Multiply the first equation through by 2 and add to the second

$$\begin{aligned} 6x + 4y &= -6 \\ 5x - 4y &= 17 \\ 11x + 0 &= 11 \end{aligned}$$

So $x = 1$. Substituting that into the original first equation yields, $3 + 2y = -3 \Leftrightarrow y = -6$ so the solutions is $(x, y) = (1, -3)$

4. A telephone bill totals \$73.20 for basic service charges and long distance charges. If the basic service charges are \$32.78 more than the long distance charges, how much of the bill was for basic service? Use algebra to solve the problem.

Solution: Let $x =$ the cost of basic service and $y =$ the cost of long distance charges. Then

$$\begin{aligned} x + y &= 73.2 \\ x &= y + 32.78 \end{aligned}$$

Substituting from the second equation into the first, we have $(y+32.78)+y = 73.2 \Leftrightarrow 2y = 40.42$ so that $y = 20.21$. Thus the basic service cost is $x = 52.99$.

5. The sum of the interior angles of a triangles is $180°$. Given that $\angle B$ is 5 more than $\angle C$ and furthermore, $\angle A$ is three times the sum of $\angle B$ and $\angle C$, set up a system of three equations in three unknowns to find the degree measures of angles A, B and C. Solve the system find the degree measures of the angles.

Solution: From the given information we set up three equations in A, B and C:

$$\begin{aligned} A + B + C &= 180 \\ B &= C + 5 \\ A &= 3(B + C) \end{aligned}$$

Substituting for B from the second equation into the first and third, we have

$$\begin{aligned} A + 2C &= 175 \\ A &= 3(2C + 5) \end{aligned}$$

Substituting from the second of these to the first, we have $3(2C + 5) + 2C = 175 \Leftrightarrow 8C = 160$ so $C = 20°$ which means that $B = 25°$ and $A = 135°$.

6. (10 points) Solve the system by back-substitution:

$$\begin{aligned} 3x - 4y + 5z &= 2 \\ 7y - 4z &= 26 \\ 5z &= -15 \end{aligned}$$

Solution: From the last equation, $z = -3$. Substituting this into the next to last equation yields $7y+12 = 26 \Leftrightarrow y = 2$. Substituting these values into the first equation yields $3x-8-15 = 2 \Leftrightarrow x = \frac{25}{3}$.

7. Consider the system of three linear equations in x, y and z:

$$\frac{1}{3}x + \frac{2}{5}y = z$$
$$x + \frac{1}{5}y = z + 1$$
$$2x - z = \frac{1}{5}y$$

(a) Clear the fractions from each equation.
Solution:

$$5x + 6y = 15z$$
$$5x + y = 5z + 5$$
$$10x - 5z = y$$

(b) Write the system in standard form.
Solution:

$$5x + 6y - 15z = 0$$
$$5x + y - 5z = 5$$
$$10x - y - 5z = 0$$

(c) Solve the system.
Solution: Eliminating x from the first two equations and then again from the first and third equations yields the 2x2 system,

$$5y - 10z = -5$$
$$13y - 25z = 0$$

Multiplyinh the first equation through by 5 and second by -2 yields

$$25y - 50z = -25$$
$$-26y + 50z = 0$$

so that $-y = -25 \Leftrightarrow y = 25$. Substituting this for y in the $5y - 10z = -5 \Leftrightarrow y - 2z = -1$ we have $25 - 2z = -1 \Leftrightarrow z = 13$. Finally, substituting for y and z in $5x + y - 5z = 5$ we have $5x + 25 - 65 = 5 \Leftrightarrow 5x = 45 \Leftrightarrow x = 9$. So the solution is $(x, y, z) = (9, 25, 13)$.

(d) Check your answer. Does it work?
Solution: Yes. Plugging these values into the original equation yields all true statements: $\frac{9}{3} + \frac{50}{5} = 13$, $9 + \frac{25}{5} = 13 + 1$, and $18 - 13 = \frac{25}{5}$.