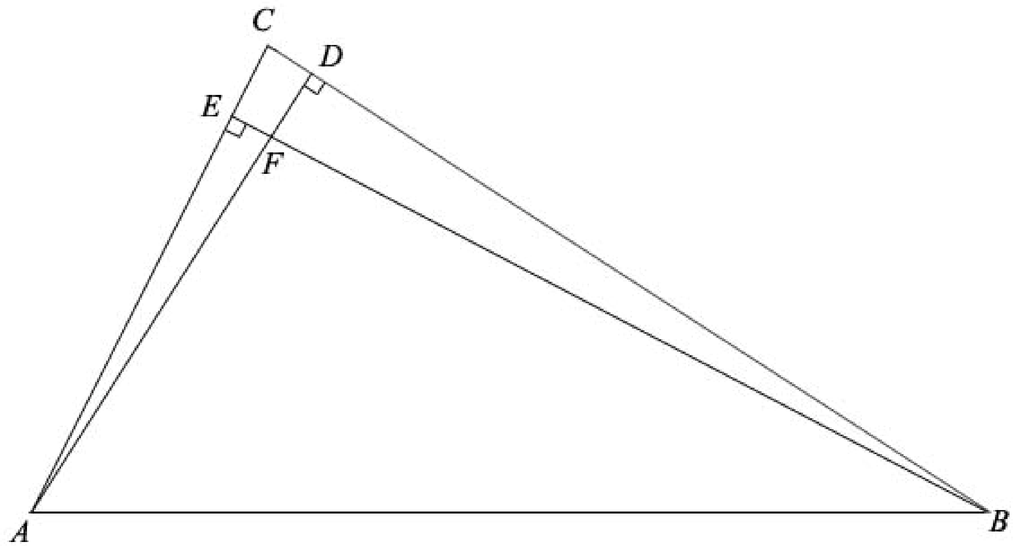
Proofs by contradiction

1. Theorem: In a scalene triangle, the altitudes are unequal.

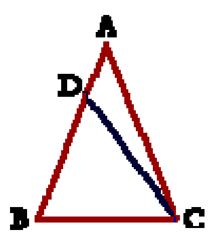


Let ΔABC be scalene. Assume that the altitudes $AD = BE$. Then, since $\angle AFE = \angle BFD$ (these are vertical angles) we have by AAS that $\Delta AFE = \Delta BFD$. Thus $\angle FAE = \angle FBD$ and $AF = BF$ by CPCTC. Now ΔBFA meets the definition of an isosceles triangle so that the base angles, $\angle FAB = \angle FBA$. This means that $\angle EAB = \angle EAF + \angle FAB = \angle DBF + \angle FBA = \angle DBA$. But if two angles of a triangle are equal then the triangle is isosceles, so ΔABC be isosceles, contradicting the fact that it's scalene. Thus the assumption that $AD = BE$ is false.

2. Euclid, Proposition 6 :

If two angles of a triangle are equal, then the sides opposite them will be equal.

Proof: Let ABC be a triangle in which angle ABC is equal to angle ACB; then side AB will equal side AC.



For if AB is not equal to AC, then one of them is longer. Let AB be longer, and from AB cut off DB equal to AC, (I. 3) which is shorter; and draw DC. (Postulate 1) Then, since we have constructed DB equal to AC, and BC is common to triangles DBC, ACB, the two sides DB, BC are equal to the two sides AC, CB respectively; and angle DBC is equal to angle ACB; (Given) therefore triangles DBC, ACB are equal in all respects, (S.A.S.) the smaller to the larger -- which is absurd. (Axiom 5) Therefore the assumption that AB is not equal to AC is false; that is, it is equal to it. Therefore, *if two angles of a triangle are equal, then the sides opposite them will be equal.* Q.E.D