Math 1B **Name (Print):** ______________________

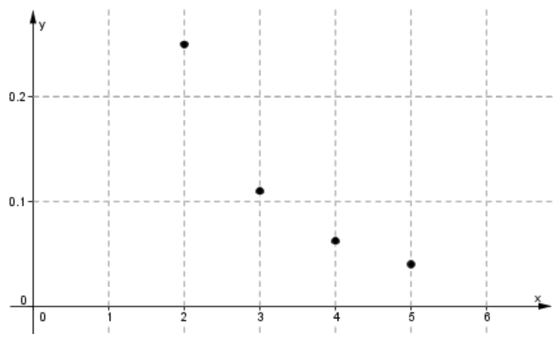
Chapter 11 Test

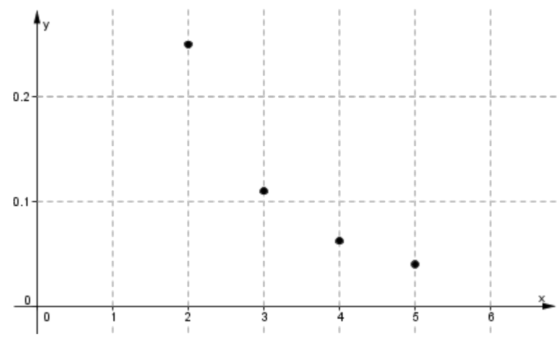
Write all responses on separate paper. Show your work in detail for credit.

1. (14 points) Find the value of c if $\sum\limits_{n=0}^{\infty} e^{-cn} = 2$.

2. (18 points) Consider the series $S = \sum\limits_{n=3}^{\infty} \frac{1}{n^2}$.

   (a) Sketch a diagram (you can use the starter plot below, or make your own) to justify the inequality, $S < \int\limits_2^{\infty} \frac{dx}{x^2}$





   (b) Make another diagram (or use the other above) to show that $\int\limits_3^{\infty} \frac{dx}{x^2} < S$

   (c) We know that $\zeta(2) = \sum\limits_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493$. Use this to determine whether $\int\limits_3^{\infty} \frac{dx}{x^2}$ or $\int\limits_2^{\infty} \frac{dx}{x^2}$ is closer to S.

3. (16 points) (a) Use the limit comparison test to prove that $\sum\limits_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ is divergent. *Hint: harmonic series.*

   (b) Prove that $\sum\limits_{n=0}^{\infty} (-1)^n \tan\left(\frac{1}{n+1}\right)$ is convergent. How would you choose N so that $R_N = S - S_N < 0.001$?

4. (24 points) $f(x) = \cos(\pi x)$.

   (a) Write the Maclaurin series for $f(x)$.

   (b) Use the ratio test to find the radius of convergence for the series.

   (c) Find the approximating Taylor polynomial of degree 4 for $g(x) = e^x \cos(\pi x)$.
   Hint: multiply two Taylor series.

5. (12 points) Use Maclaurin series to justify that $e^{\imath x} = \cos(x) + \imath \sin(x)$. Recall that $\imath^2 = -1$.

6. (16 points) Recall that the binomial series has

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$$

   (a) Find the cubic Taylor polynomial approximating $f(x) = \frac{1}{(1+x)^3}$.

   (b) Consider $\sqrt[3]{1729} = \sqrt[3]{1728+1} = 12\sqrt[3]{1 + 1/1728}$ and use the binomial series for $(1+x)^{1/3}$ to approximate this to the nearest millionth. (You can leave your expression in fractional form.)