Math 1B **Name (Print):** ____________________

Final Exam

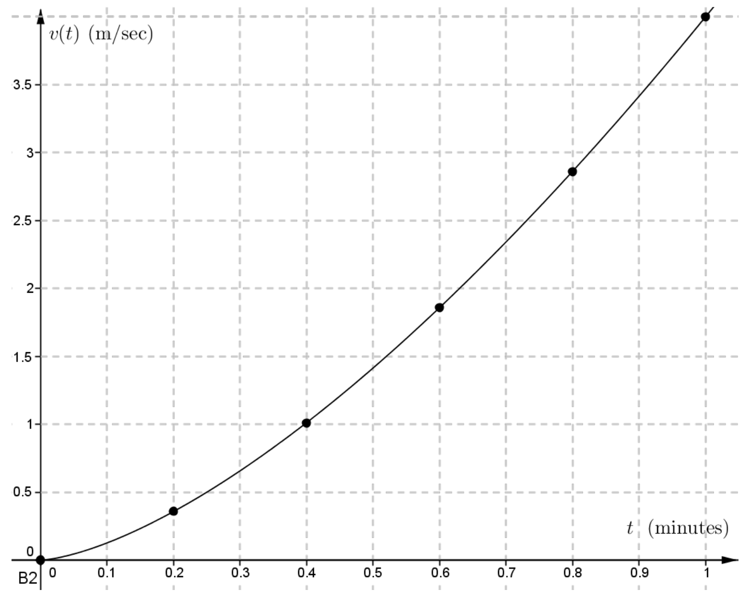
Write all responses on separate paper. Show your work in detail for credit.

There are 9 problems from which you will choose 8. Indicate here which problem you'd like to omit: ☐

1. The graph below plots the velocity of a bicycle accelerating downhill over a period of one minute. Points from the graph are tabulated here:

t (min)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
v (m/sec)	0	0.13	0.36	0.66	1.01	1.41	1.86	2.34	2.86	3.42	4.00

Approximate the distance traveled by the bicyclist during this minute $[0, 30]$ with



   (a) 5 subintervals of equal lengths and right endpoints as sample points.

   (b) 5 trapezoids of equal widths.

   (c) 5 subintervals of equal lengths and midpoints as sample points.

   (d) Explain the relationship between the trapezoidal sum and the midpoint sum in terms of the concavity of the curve.

2. Evaluate:

   (a)
$$\int_0^9 \frac{d}{dx}\sqrt{271 + x^3}\; dx$$

   (b)
$$\frac{d}{dx}\int_0^{\cos(x)} \sin^{-1} t \, dt$$

3. Use integration by parts and substitution methods to evaluate the integral.
Recall that $\frac{d}{dy}\sin^{-1} y = \dfrac{1}{\sqrt{1-y^2}}$

   (a) $\int_0^1 \sin^{-1} y \, dy$

$u =$	$dv =$
$du =$	$v =$

   (b) $\int_0^1 \left(\sin^{-1} y\right)^2 \, dy$

$u =$	$dv =$
$du =$	$v =$

Use your own space to make your tables for parts and substitution, this is here just to remind you to do that.

4. Consider the integral function,

$$I_n(t) = \int_0^t \sec^n(x)\,dx = \int_0^t \sec^{n-2}(x)\sec^2(x)\,dx$$

   (a) Use integration by parts to show that

$$I_n(t) = \frac{\tan t \sec^{n-2} t}{n-1} + \frac{n-2}{n-1} I_{n-2}(t)$$

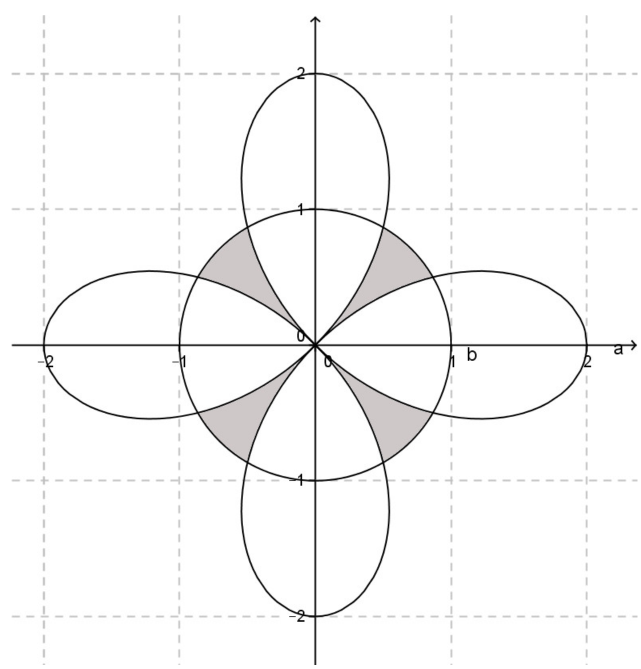
   Make an table of u, dv, du, and v. Look for the integral recurring after applying a trig identity.

   (b) Use the reduction formula from (a) to evaluate $\int_0^{\pi/4} \sec^4(x)\,dx$

5. The region bounded by the x-axis and the curve $y = x\sin x$ between $x = 0$ and $x = \pi$ is revolved around the y-axis. Find the volume generated.

6. The ellipse bounded by the parametric equations $\begin{array}{c} x = \cos t \\ y = 2\sin t \end{array}$ is revolved around the y-axis and the volume generated is filled with water. Set up and evaluate an integral to find the minimum work required to pump the water out through a hole in the top. Recall that the force density of water is approximately 9800 Newtons per cubic meter.

7. Find the area of the region outside the polar curve $r = 2\cos 2\theta$ and inside the unit circle, as shaded in the diagram.



8. Use a Maclaurin series to approximate each to the nearest billionth (10^{-9}).

   (a) $\sin(0.1)$

   (b) $\sqrt[3]{1.1}$ Recall that $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$

9. Find the 6th degree Taylor polynomial approximation for $\sin^2(x)$ in two ways. These should be the same.

   (a) by squaring the Taylor polynomial for $\sin(x)$

   (b) by using $\sin^2 x = \dfrac{1-\cos 2x}{2}$ and expanding $\cos 2x$ in a Taylor polynomial.