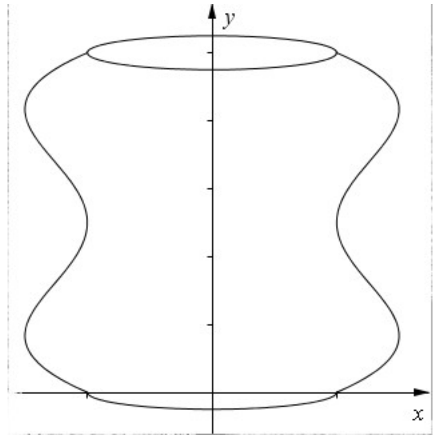
Math 1B **Name (Print):** ____________________

Chapter 7 Test

Write all responses on separate paper. Show your work for credit. Do not use a calculator.

1. (20 points) The curve $x = 2\sin(\pi y) + \cos(2\pi y)$ for $0 \le y \le 1$ generates the volume depicted at right when revolved about the $y-$axis. If the volume is filled with water, find the minimum work required to pump the water out through the top. Assume all units are MKS (meter, kilogram, second) and that the weight density of water is 9800 Newtons per cubic meter.
   These identities are useful in the integral:
   $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$, $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$,
   $2\sin(\alpha)\cos(\beta) = \sin(\alpha-\beta) + \sin(\alpha+\beta)$



2. (20 points) Derive a reduction formula for $I_n = \displaystyle\int x^n e^{ax}\,dx$ in terms of I_{n-1}.

   Use the reduction formula to evaluate $\displaystyle\int_0^1 x^4 e^{ax}\,dx$

3. Let

$$U(x) = \begin{cases} 0 & : x < 0 \\ 1 & : x \ge 0 \end{cases}$$

   and then define $\delta(x) = U'(x)$

   (a) Show that $\displaystyle\int_{-A}^{A} \delta(x)\,dx = 1$.

   (b) Use integration by parts to prove that for any function $v(x)$, $\displaystyle\int_{-A}^{A} v(x)\delta(x)\,dx = v(0)$.

4. (20 points) Evaluate each integral:

   (a) $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+2x+2}}\,dx$

   (b) $\displaystyle\int_2^3 \frac{5}{(x-1)(x^2+2x+2)}\,dx$

5. (20 points) Determine whether each integral is convergent or divergent. If it's convergent, evaluate it.

   (a) $\displaystyle\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx$

   (b) $\displaystyle\int_0^1 \frac{5}{\sqrt{1-x^2}}\,dx$