

Write all responses on separate paper. Show your work in detail for credit. Do not copy other's work.

1. (16 points) Consider the curve defined by the parametric equations

$$\begin{aligned}x &= \sin^3(t) \\ y &= \cos(t)\end{aligned}$$

- (a) Complete the table of values and extend it as needed to sketch a complete graph for the the curve in the  $xy$ -plane.

$t$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$x$					
$y$					

- (b) Find the coordinates of the points of inflection.

2. (16 points) Consider the parametric equations,

$$x(t) = \int_0^t \cos(u^2) du, \quad y(t) = \int_0^t \sin(u^2) du$$

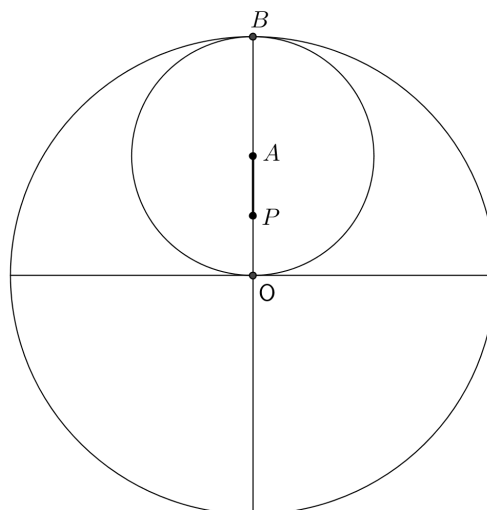
- (a) Find the the arclength of this curve over the interval from  $t \in [0, s]$ .  
 (b) For what values of  $t$  is the line tangent to the curve vertical?

3. (16 points) Consider the parametric equations

$$\begin{aligned}x(\theta) &= 2 \tan \theta \\ y(\theta) &= 2 \cos \theta\end{aligned}$$

- (a) Make a table of values and sketch a graph for the curve.  
 (b) Set up and simplify an integral for the surface area generated when the part of the curve  $t \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$  is revolved about the  $x$ -axis.

4. (20 points) In the diagram, a circle of radius  $OA = 2$  rolls around the inside of a circle of radius  $OB = 4$  with the half radius  $AP = 1$  originally positioned so that  $P$  is on  $OB$ . Find a parameterization for the path that  $P$  traces out as the inner circle rolls (without slipping.) Start by choosing an appropriate parameter. Is that a familiar curve?



5. (16 points) Find the area enclosed by one of the loops in the graph of the polar function  $r = 2 \sin \theta - \tan \theta$ .  
 6. (16 points) Use Simpson's rule with  $n = 6$  to approximate the arc length of  $r = \cos(4 \cos(\theta))$  on the interval  $t \in \left[0, \frac{\pi}{2}\right]$ .