Math 1B **Name (Print):** ____________________

Chapter 5&§6.1 Test

Write all responses on separate paper. Show your work for credit. Do not use a calculuator.

1. (15 points) Recall the definition for the **area** A of the region under a graph of a continuous function as the limit of the sum of areas of approximating rectangles:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

We will use this to set up the computation for the area under $f(x) = \sin^2 x$ above $\left[0, \frac{\pi}{2}\right]$.

(a) Draw a diagram illustrating this region.

(b) Express Δx in terms of the number of intervals, n, in the partition of $\left[0, \frac{\pi}{2}\right]$.

(c) Express the ith point in the partition, x_i, in terms of i and n.

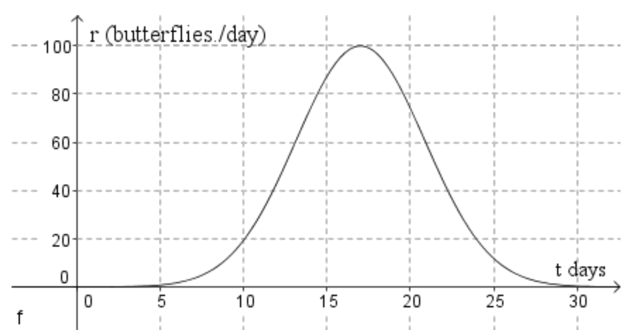
(d) Express the area as a limit involving only the variables n and i.

(e) Use the identity $\sin^2 x = \dfrac{1-\cos(2x)}{2}$ and the Fundamental Theorem of Calculus to simplify the value of the area.

2. (10 points) The graph below shows shows the rate of butterfly births in a Monarch butterfly nest over a period of thirty days.

(a) Approximate the area under the curve using a partition of $[0, 30]$ with 3 sub-intervals of equal length and midpoints as sample points. Approximate the function values from the graph.

(b) Explain what the integral $\int_0^{30} f(t)\,dt$ means in terms of the function $r = f(t)$.



3. (12 points) The speed, $v(t)$, in meters per second of a roller skater as a function of time, t, measured in seconds, is tabulated below:

t (sec)	0	2	4	6	8	10	12
v (m/s)	-3	-2	-0.5	0.5	1	3	3.5

(a) Use these data to compute $R_6 = \sum_{i=1}^{6} v(t_i)\Delta t$.

(b) Use the data to compute L_6.

(c) Use the data to compute M_3.

(d) Describe what these sums represent in terms of the roller skater.

4. (15 points) Use the Fundamental Theorem of Calculus Part I to compute the derivative of the given functions:

(a) $g(x) = \displaystyle\int_{-1}^{x} e^{-t^2}\,dt$

(b) $F(x) = \displaystyle\int_{x}^{0} \arctan t^2\,dt$

(c) $h(x) = \displaystyle\int_{1}^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$

(d) $p(x) = \displaystyle\int_{\cos x}^{\sin x} t^2\,dt$

(e) $c(x) = \displaystyle\int_{0}^{1} \cos(t^2)\,dt$

5. (12 points) Consider the integral $\displaystyle\int_1^5 2x - x^2\,dx$.

   (a) Use the definition of the integral to evaluate it as a limit of Riemann sums. Be sure to state what Δx and x_i are.

   (b) Use the Fundamental Theorem of Calculus to verify your result.

6. (12 points) Compute each limit by interpreting it as a definite integral.

   (a) $\displaystyle\lim_{n\to\infty} \frac{3}{n}\sum_{i=0}^{n} \cos\left(1 + \frac{3i}{n}\right)$

   (b) $\displaystyle\lim_{n\to\infty} \frac{4}{n}\sum_{i=0}^{n} \exp\left(1 + \frac{4i}{n}\right)$

7. (12 points) Evaluate each integral.

   (a) $\displaystyle\int_0^1 \frac{z^2}{z^3+1}\,dz$

   (b) $\displaystyle\int_0^a x\sqrt{a^2 - x^2}\,dx$

   (c) $\displaystyle\int_{-\pi/2}^{\pi/2} x^4 \sin(x^3)\,dx$ *Hint*: use a symmetry argument for this.

8. (12 points) Sketch the region enclosed by the curves $y = \cos x, y = 2 - \cos x, 0 \le x \le 2\pi$ and compute its area.