Math 1B-Spring 2017 **Name (Print):** ______________________

Final Exam

Write responses on this paper, or attach additional paper, as needed. Show your work in detail for credit. Do not use a calculator.

1. Here we will use the infinitesimal approximation, $ds^2 = dx^2 + dy^2$ to estimate the length of the sine curve $y = \sin x$, for $0 \leq x \leq \pi$.

   (a) Complete the table of values for x and $\dfrac{ds}{dx} \approx \sqrt{1 + (dy/dx)^2}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	$\sqrt{2}$	$\frac{\sqrt{7}}{2}$					

   (b) Use the tabulated values from (a) to express the right endpoint approximating sum on three subintervals,
   $$\int_0^{\pi} \sqrt{1 + (dy/dx)^2}dx \approx R_3 = \Delta x \sum_{i=1}^{3} \sqrt{1 + \cos^2(x_i)}$$ Don't approximate radicals.

   (c) Use the tabulated values from (a) to express the left endpoint approximating sum on three subintervals,
   $$\int_0^{\pi} \sqrt{1 + (dy/dx)^2}dx \approx L_3 = \Delta x \sum_{i=1}^{3} \sqrt{1 + \cos^2(x_{i-1})}$$

   (d) Find the approximating Simpson sum on six subintervals, S_6

2. Find the average value of $f(x) = \dfrac{1}{1 + x^2}$ on $[0, \pi/4]$. Recall that $f_{\texttt{avg}} = \dfrac{1}{b-a} \displaystyle\int_a^b f(x)\, dx$

3. Each given function is continuous on the given interval, so the Mean Value Theorem for integrals applies on the interval. Find all values of c guaranteed by the Mean Value Theorem for Integrals. That is, all c such that, $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$.

   (a) $f(x) = \sin x$ on $[0, \pi]$.

   (b) $f(x) = 4x^3 + 10x$ on $[0, 3]$.

4. Use integration by parts and substitution methods to evaluate the integral. Indicate on the paper what your initial parts are for each.

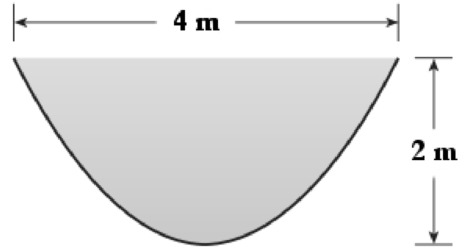
   (a) $\int_0^{\ln 2} \frac{x}{e^x}\,dx$

$u =$	$dv =$
$du =$	$v =$

   (b) $\int_0^{\pi/2} \frac{x}{\sec x}\,dx$

$u =$	$dv =$
$du =$	$v =$

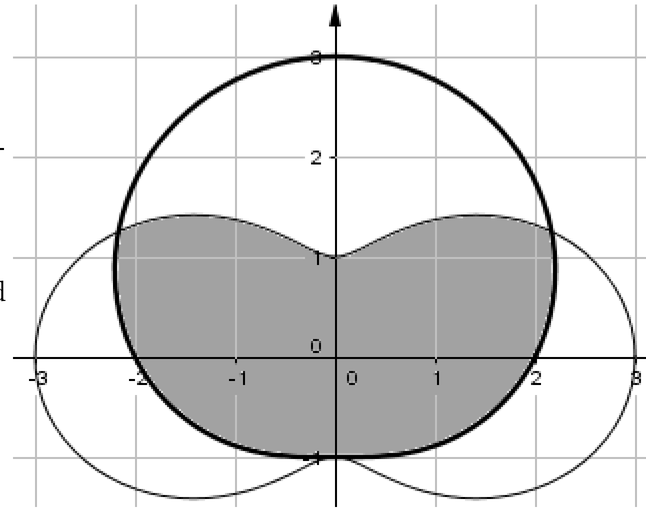
5. A trough is filled with water and its vertical ends have the shape of the parabolic region in the figure. Find the hydrostatic force on one end of the trough. Note: the force density of water is $\approx 9800\mathtt{N/m}^3$.



6. Consider the region bounded by $y = x$ and $y = \sqrt{x}$

(a) Find the volume generated by rotating this region about the x-axis using the washer method.

(b) Find the volume generated by rotating this region about the y-axis using the shell method (different volume!)



7. Consider the problem of finding area by integrating with polar coordinates.

(a) Explain why the area enclosed by a curve $r = f(\theta)$ for $\theta_1 \leq \theta \leq \theta_2$ is given by $\int_{\theta_1}^{\theta_2} \frac{(f(\theta))^2}{2}\, d\theta$. Draw a diagram and describe the relevant formula.

(b) Find the area of the region inside the polar curve $r = 2 + \cos 2\theta$ and inside the curve $r = 2 + \sin\theta$, as shaded in the diagram.

8. Use a Maclaurin series to approximate $\sqrt{1.2}$ to the nearest millionth (10^{-6}). Recall that $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$

9. Use the Taylor series, $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ to express $f(x) = x^3 + x + 1$ as a sum of multiples of powers of $x - 1$.