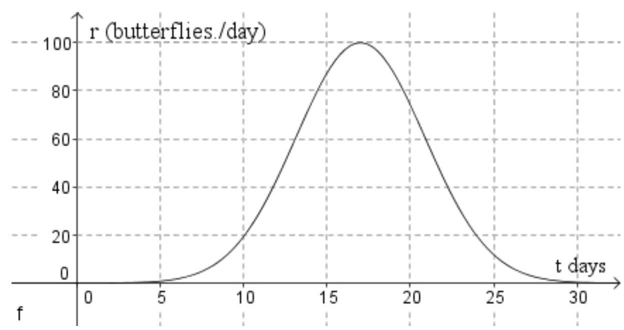
Math 1B **Name (Print):** ______________________
Chapter 5 Test

Write all responses on separate paper. Show your work for credit. Do not use a calculuator.

1. (18 points) Consider the area bounded by $f(x) = 1 - (x-1)^2$ and the x-axis.
   (a) Draw a diagram illustrating this region.
   (b) Approximate the area using a partition with 3 intervals of equal length and midpoints as sample points.
   (c) Use the definition of the definite integral to compute the area as the limit of a Riemann sum. Don *not* use the Fundamental Theorem of Calculus.

2. (12 points) The graph below shows shows the rate of butterfly births in a Monarch butterfly nest over a period of thirty days.
   (a) Approximate the area under the curve using a partition of $[0, 30]$ with 3 subintervals of equal length and midpoints as sample points. Approximate the function values from the graph.
   (b) Explain what the integral $\int_0^{30} f(t)\,dt$ means in terms of the function $r = f(t)$.



3. (15 points) The speed, v, of a runner is measured at various times, t, to produce the tabulated values:

t (sec)	0	1	2	3	4	5	6
v (m/s)	1	3	5	6	7	8	8

   (a) Approximate the distance the runner has traveled in these six seconds using three subintervals of equal length and right endpoints as sample points.
   (b) Approximate the distance the runner has traveled in these six seconds using three subintervals of equal length and left endpoints as sample points.
   (c) Approximate the distance the runner has traveled in these six seconds using three subintervals of equal length and right midpoints as sample points.

4. (21 points) Evaluate:
   (a)
$$\int_0^{\pi} \frac{d}{dx} \sin x^2 \; dx$$
   (b)
$$\frac{d}{dx} \int_0^{x^2} \sin t^2 \; dt$$
   (c)
$$\frac{d}{dx} \int_0^{\pi} \sin t^2 \; dt$$

5. (18 points) Evaluate the integral. If you use a substitution, be explicit about the values of u and du.

(a)

$$\int_1^e \frac{\sin(\ln x)}{x}\,dx$$

(b)

$$\int_{\cos(x)}^{\sin(x)} t\,dt$$

6. (16 points) Compute each limit by interpreting it as a definite integral.

(a) $\lim_{n\to\infty} \frac{3}{n}\sum_{i=0}^{n} \cos\left(1+\frac{3i}{n}\right)$

(b) $\lim_{n\to\infty} \frac{4}{n}\sum_{i=0}^{n} \exp\left(1+\frac{4i}{n}\right)$