CLASSROOM CAPSULES

Classroom Capsules are short (1–3 page) notes that contain new mathematical insights on a topic from undergraduate mathematics, preferably something that can be directly introduced into a college classroom as an effective teaching strategy or tool. Classroom Capsules should be prepared according to the guidelines on the inside front cover and sent to any of the above editors.

Forest Carbon Uptake and the Fundamental Theorem of Calculus

John Zobitz (zobitz@augsburg.edu), Augsburg College, Minneapolis, MN

The fundamental theorem of calculus connects net change in a quantity to its rate of change. A typical classroom application is to determine net distance traveled from velocity. The fundamental theorem can also be used to investigate the dynamic nature of terrestrial ecosystems and their role in the global carbon cycle, an application that is just as accessible to calculus students as distance, and highly relevant today.

Figure 1 shows the diurnal average net ecosystem carbon exchange (NEE) between a terrestrial ecosystem and the atmosphere during July for a high-elevation coniferous forest [**6**]. Net ecosystem exchange is a *rate* (measured in grams of carbon per m^2 per

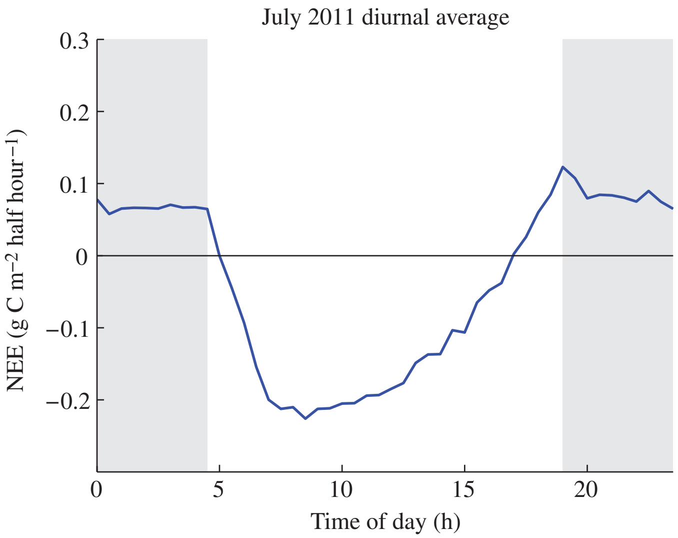

Figure 1. Diurnal average net ecosystem carbon exchange (NEE) for a high-elevation coniferous forest for July 2011. Positive values represent a net flux of carbon to the atmosphere. Gray shading represents nighttime values.

---

http://dx.doi.org/10.4169/college.math.j.44.5.421
MSC: 62P12, 92B05

half hour) that represents the net biosphere-atmosphere exchange of carbon per unit area of an ecosystem, or the net carbon flux per unit time.

Measurement of NEE derives from principles of mass conservation [**1, 2**] and requires knowledge of micrometeorology and environmental biophysics. Diurnal variation in NEE is due to a variety of factors. Biological processes such as photosynthesis (uptake of carbon from the atmosphere by plants) and respiration (release of carbon to the atmosphere) all influence the measured value of NEE during the day. Photosynthesis is light dependent reaction, so typically NEE is negative during daylight. Other processes affecting net ecosystem carbon exchange are seasonal transitions (phenology) and the weather. NEE, along with supplementary measurements and mathematical models, is used to estimate ecosystem photosynthesis and components of ecosystem respiration [**8**].

NEE is the rate of carbon exchange. The fundamental theorem says that net carbon uptake is the cumulative integral of NEE across time:

$$\text{cumulative net carbon uptake} = \int_0^t \text{NEE}(x)\, dx.$$

In other words, the signed area underneath the curve in Figure 1 is the total amount of carbon taken up by an ecosystem over the course of a day. Using numerical integration (a left endpoint Riemann sum), this value is approximately -1.80 grams of carbon per m^2, showing that the ecosystem is a net absorber of carbon in the summer.

After a class develops this understanding of net ecosystem exchange, I display the diurnal average during January (shown in Figure 2). Careful inspection of the vertical axis shows that the net ecosystem carbon exchange is not zero. During January, this coniferous ecosystem is cold and has a dense snowpack. The primary biological activity during this time period is respiration, so the measured NEE is positive. The total signed area is approximately 0.40 grams of carbon per m^2, so the ecosystem is a small, net carbon source in the winter. Contrasting the two months (January versus July) reveals the dynamic nature of terrestrial ecosystems.

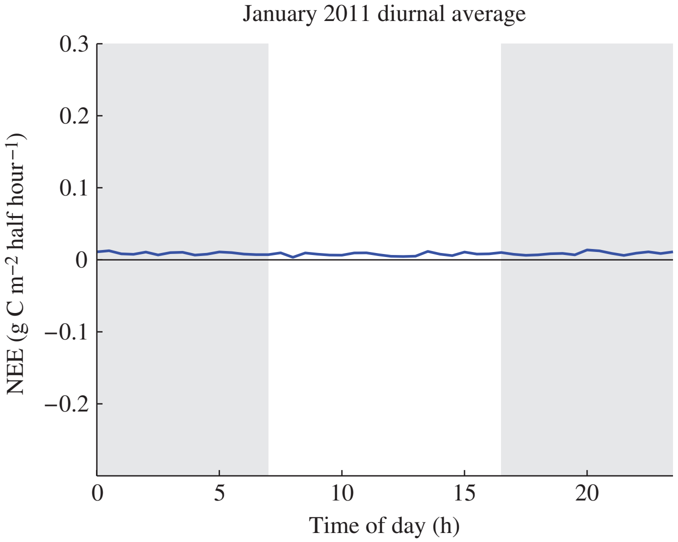

Figure 2. Diurnal average of net ecosystem carbon exchange for a high-elevation coniferous forest in January.

A network of over 500 sites measure NEE worldwide [**7**]. Some of these sites have been established for longer than a decade. There is an astounding amount of data to analyze. Many of these sites allow open access to a portion of their data [**4**]. For a longer term research project, my students access these data and apply the fundamental theorem to determine the annual carbon uptake for various ecosystems. Some examples are shown in Figure 3. Niwot Ridge is the site whose data are shown in Figures 1 and 2. Collelongo is a deciduous Mediterranean forest [**3**]. Black Spruce is a Canadian boreal coniferous forest that had a significant burn in 1994. This annual pattern of carbon uptake suggests forest regrowth from the fire [**5**].

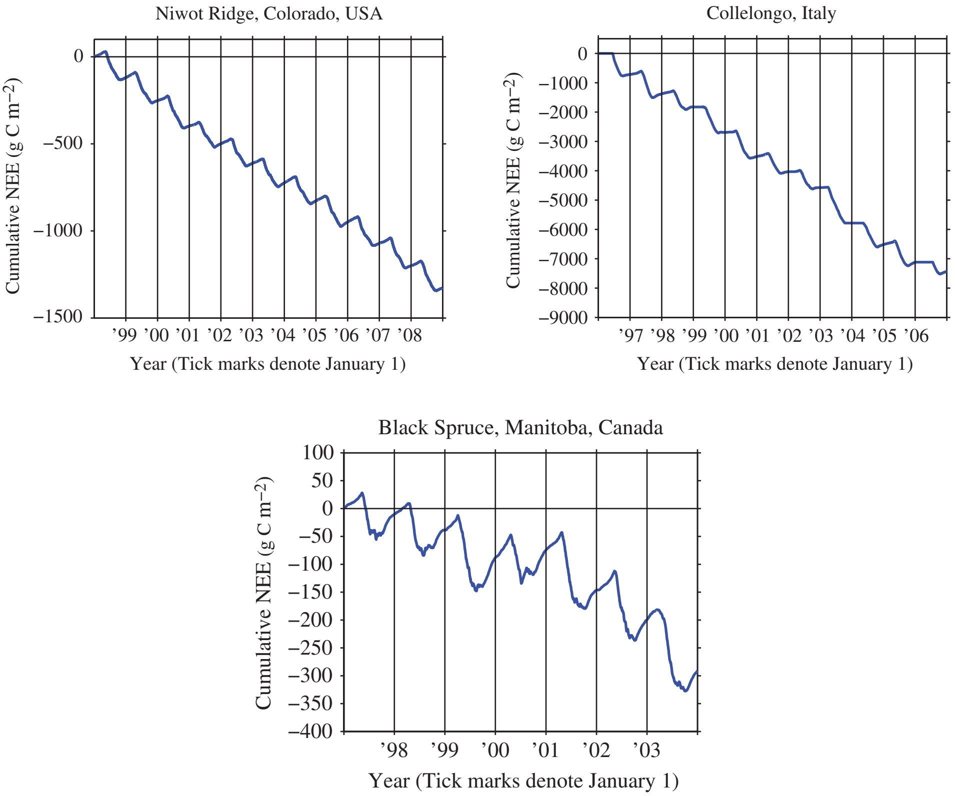

Figure 3. Cumulative net carbon uptake for a high-elevation coniferous forest (Niwot Ridge), a deciduous Mediterranean forest (Collelongo), and a boreal coniferous forest (Black Spruce).

These three sites illustrate the variation in carbon uptake across ecosystems. The negative values show that all the ecosystems are net carbon sinks, and the trend is toward increasing absorption of carbon. Analysis of NEE helps scientists understand how climate change affects ecosystem carbon uptake locally, and provides context for understanding carbon uptake globally.

Conclusion. Determining ecosystem carbon uptake is a stimulating case study in the application of the fundamental theorem of calculus and numerical integration. Because the data come from current research and are accessible to the public, students are exposed to the interdisciplinary nature of science, and gain more insight into the mathematics of planet Earth.

Acknowledgment. Many thanks to Russ Monson, Peter Blanken, PI at Niwot Ridge, Ricardo Valentini, PI at Collelongo, Brian Amiro, and PI at Black Spruce for use of their data. Thanks to Jazmine Darden and Nana Owusu for their hard work during a undergraduate summer research project investigating net carbon uptake.

This work used open access NEE data acquired by the FLUXNET community. The policy and acknowledgment for this dataset can be found at: `http://www.fluxdata.org/Shared%20Documents/Policy_Opened_Final.pdf`.

Summary. Using the fundamental theorem of calculus and numerical integration, we investigate carbon absorption of ecosystems with measurements from a global database. The results illustrate the dynamic nature of ecosystems and their ability to absorb atmospheric carbon.

References

1. D. Baldocchi, E. Falge, L. Gu, R. Olson, D. Hollinger, S. Running, P. Anthoni, C. Bernhofer, K. Davis, R. Evans, J. Fuentes, A. Goldstein, G. Katul, B. Law, X. Lee, Y. Malhi, T. Meyers, W. Munger, W. Oechel, K. T. P. U, K. Pilegaard, H. P. Schmid, R. Valentini, S. Verma, T. Vesala, K. Wilson, and S. Wofsy, FLUXNET: A New Tool to Study the Temporal and Spatial Variability of Ecosystem–Scale Carbon Dioxide, Water Vapor, and Energy Flux Densities, *Bulletin of the American Meterological Society* **82** (2001) 2415–2434; available at `http://dx.doi.org/10.1175/1520-0477(2001)082<2415:FANTTS>2.3.CO;2`.
2. D. Baldocchi, M. Reichstein, D. Papale, L. Koteen, R. Vargas, D. Agarwal, and R. Cook, The role of trace gas flux networks in the biogeosciences, *Eos Transactions* **93** (2012) 217–218; available at `http://dx.doi.org/10.1029/2012EO230001`.
3. N. Carvalhais, M. Reichstein, G. J. Collatz, M. D. Mahecha, M. Migliavacca, C. S. Neigh, E. Tomelleri, A. A. Benali, D. Papale, and J. Seixas, Deciphering the components of regional net ecosystem fluxes following a bottom-up approach for the Iberian Peninsula, *Biogeosciences Discussions* **7** (2010) 4801–4855; available at `http://dx.doi.org/10.5194/bgd-7-4801-2010`.
4. Fluxnet, *Public FLUXNET Dataset Information*; available at `http://www.fluxdata.org/datainfo`.
5. M. L. Goulden, A. M. S. Mcmillan, G. C. Winston, A. V. Rocha, K. L. Manies, J. W. Harden, and B. P. Bond-Lamberty, Patterns of NPP, GPP, respiration, and NEP during boreal forest succession, *Global Change Biology* **17** (2011) 855–871; available at `http://dx.doi.org/10.1111/j.1365-2486.2010.02274.x`.
6. R. K. Monson, A. A. Turnipseed, J. P. Sparks, P. C. Harley, L. E. Scott-Denton, K. Sparks, and T. E. Huxman, Carbon sequestration in a high-elevation, subalpine forest, *Global Change Biology* **8** (2002) 459–478; available at `http://dx.doi.org/10.1046/j.1365-2486.2002.00480.x`.
7. Oak Ridge National Laboratory Distributed Active Archive Center (2011), available at `http://fluxnet.ornl.gov`.
8. M. Reichstein, E. Falge, D. Baldocchi, D. Papale, M. Aubinet, P. Berbigier, C. Bernhofer, N. Buchmann, T. Gilmanov, A. Granier, T. Grnwald, K. Havránková, H. Ilvesniemi, D. Janous, A. Knohl, T. Laurila, A. Lohila, D. Loustau, G. Matteucci, T. Meyers, F. Miglietta, J.-M. Ourcival, J. Pumpanen, S. Rambal, E. Rotenberg, M. Sanz, J. Tenhunen, G. Seufert, F. Vaccari, T. Vesala, D. Yakir, and R. Valentini, On the separation of net ecosystem exchange into assimilation and ecosystem respiration: review and improved algorithm, *Global Change Biology* **11** (2005) 1424–1439; available at `http://dx.doi.org/10.1111/j.1365-2486.2005.001002.x`.

Climate Modeling in the Calculus and Differential Equations Classroom

Emek Köse (ekose@smcm.edu) and Jennifer Kunze (jckunze@smcm.edu), St. Mary's College of Maryland, St. Mary's City, MD

We introduce here the basic principles of climate science for a one-dimensional Energy Balance Model (EBM), as first proposed by Peter Imkeller [**2**]. Calculus and differential equations students can use it as a basis for further research into the climate system, particularly the ice-albedo feedback mechanism.

---

http://dx.doi.org/10.4169/college.math.j.44.5.424
MSC: 34A34, 34D20, 97M10