

DIRECTIONS: Show your work in the space provided. Please box your answers, and give units where appropriate. This exam is worth 50 points. Incorrect, incomplete, and/or missing work may not receive full credit. Good luck!

1. (#47) Evaluate the integral. (4 pts.)

$$\int \frac{x^{1/3}}{x^{8/3} + 2x^{4/3} + 1} dx$$

2. Let R be the region of the xy -plane bounded by the graphs of $y = x^3 + 7x$, $x = 2$, $x = 3$, and the x -axis. Set up, but **do not evaluate**, integrals that represent the volume of the solid generated by revolving R around

(a) the x -axis. (3 pts.)

(b) the y -axis. (3 pts.)

3. Determine the convergence or divergence of the following sequence. If the sequence converges, find its limit. Show enough work to make your reasoning clear.(4 pts.)

$$a_n = \frac{n-1}{n} - \frac{n}{n-1}, \quad n \geq 2$$

4. Determine whether the following series converges or diverges. Show enough work to make your reasoning clear.(4 pts.)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{n(n+1)}$$

5. (#9) Evaluate the integral.(5 pts.)

$$\int \frac{8dy}{y^2\sqrt{4-y^2}} dy$$

6. Water leaks from a tank at the rate of t^2e^{-t} liters per hour for t in $[0, 5]$. How much water will leak from the tank in the first two hours?(5 pts.)

7. Find the first three nonzero terms of the Taylor series for $f(x) = \ln(x)$ centered at $x = 1$. (5 pts.)

8. Solve the initial-value problem.(4 pts.)

$$yy' - e^x = 0, \quad y(0) = 4$$

9. Find the area of the region enclosed by the graphs of the functions $f(x) = 2 \sin(x)$, $g(x) = \tan(x)$, with $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$. (4 pts.)

10. Evaluate the definite integral. Give the exact answer. (5 pts.)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^3 x \cos^5 x \, dx$$

11. Determine whether the following integral is convergent or divergent. Evaluate it if it is convergent. (4 pts.)

$$\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6} \, dx$$