Math 1B – Spring '10 – Take Home Portion of Chapter 5 Test

Instructions: Write a report explaining your responses to the following questions. This is an opportunity to develop your technical writing skills. You may consult other students in preparation of your responses, but all your writing must be your own. Do not put off preparing your report since it will then likely be of poor quality.

Upper and Lower Riemann Sums

Let $f(x)$ be a positive, continuous, and increasing function on $[a,b]$. Your goal is to develop a technique to estimate the integral $\int_a^b f(x)\,dx$ within a given margin of error without evaluating the integral.

1. Your first step is to understand two special kinds of Riemann sums. Partition $[a,b]$ into *n* equal parts. What is the length of each subinterval? For the first type of Riemann sum, choose the height of each rectangle to be the maximum value of *f* over the subinterval. How do you know the maximum exists? For what value of *x* do you get the maximum value? The Riemann sum defined by taking the maximum value of *f* over each subinterval is called the *upper sum*. Explain why $\int_a^b f(x)\,dx$ is less than or equal to any upper sum. Write explicitly the upper sum when *n* = 4.
2. For the second kind of Riemann sum, repeat part (1), but replace the maximum value of *f* over each subinterval by the minimum value. This Riemann sum is called a *lower sum*. Explain why $\int_a^b f(x)\,dx$ is greater than or equal to any lower sum. Write explicitly the lower sum when *n* = 4.
3. Let U_4 be the upper sum and L_4 be the lower sum for *n* = 4 found above. The difference $U_4 - L_4$ is non-negative. Why? Simplify $U_4 - L_4$ as much as possible. Next find a simplified expression for $U_n - L_n$.
4. Part (3) gives you a way to make the upper and lower sums as close to each other as you wish. Suppose ε is a given positive real number. How can you use part (3) to estimate $\int_a^b f(x)\,dx$ with a margin of error of at most ε? Explain.
5. Estimate $\int_{0.5}^{2} \left(1+x^2\right)^{1/3} dx$ with an error of at most 10^{-1} using the ideas of part (4).
6. Can you think of other ways (other than the maximum and minimum) to choose the value of *f* on each subinterval that would give better approximations to the integral than U_n or L_n? Discuss.