Math 1B Take-home problems for Test 3 – spring '10

Consider the following three theorems (note ξ is the Greek letter "xi" which is pronounced "zai"):

Theorem 1: If f is a continuously differentiable function on [a, b] for which $f''(u)$ exists at each point u of (a, b) and if T_n is the n-subdivision trapezoidal rule approximation to $\int_a^b f(t)\,dt$, then there exists ξ in (a, b) such that

$$\int_a^b f(t)\,dt = T_n - f''(\xi)\frac{(b-a)^3}{12n^2}$$

Theorem 2: If f is a continuously differentiable function on [a, b] for which $f''(u)$ exists at each point u of (a, b) and if M_n is the n-subdivision midpoint rule approximation to $\int_a^b f(t)\,dt$, then there exists ξ in (a, b) such that

$$\int_a^b f(t)\,dt = M_n + f''(\xi)\frac{(b-a)^3}{24n^2}$$

Theorem 3: If f is a continuously differentiable function on [a, b] for which $f''(u)$ exists at each point u of (a, b) and if S_{2n} is the $2n$-subdivision Simpson rule approximation to $\int_a^b f(t)\,dt$, then there exists ξ in (a, b) such that

$$\int_a^b f(t)\,dt = S_{2n} - f^{(4)}(\xi)\frac{(b-a)^5}{180(2n)^4}$$

These theorems are "existence" theorems in that they don't say how to find zeta, just that such a ξ exists. Give detailed explanations as to how to find ξ for all three theorems for each of the following integrals and values of n. Use a computing device only sparingly, as needed:

1. $\int_0^1 x^4\,dx$ for $n = 2$.
2. $\int_0^1 x^5\,dx$ for $n = 2$.
3. $\int_0^1 x^4\,dx$ for $n = 4$.
4. $\int_0^1 x^5\,dx$ for $n = 4$.
5. $\int_0^1 \frac{1}{x^2+1}\,dx$ for $n = 4$.
6. $\int_0^{\pi/2} \sin^4(x)\,dx$ for $n = 4$.