# Math 1B – Approximating sums for integrals on various calculators and CAS's.

Consider the problem of approximating $\int_0^\pi \sec\left(\dfrac{x}{3}\right) dx$ by sampling $n$ evenly spaced subintervals at midpoints.

The length of each subinterval is then

$$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$$

so the right endpoint of the $i$th subinterval in the partition is

$$x_i = a + i\Delta x = 0 + \frac{i\pi}{n} = \frac{i\pi}{n}$$

This means that the midpoint of the $i$th subinterval is

$$\overline{x}_i = \frac{i\pi}{n} - \frac{\Delta x}{2} = \frac{i\pi}{n} - \frac{\pi}{2n} = \frac{(2i-1)\pi}{2n}$$

So the sum we're seeking to evaluate has the form

$$\sum_{i=1}^{n} \sec\left(\frac{\overline{x}_i}{3}\right)\Delta x = \frac{\pi}{n}\sum_{i=1}^{n} \sec\left(\frac{(2i-1)\pi}{6n}\right)$$

So let's implement this formula on the TI83.

Start by finding the command `seq` in the catalog:

```
CATALOG
 2-SampZTest(
 Scatter
 Sci
 Select(
 Send(
▶seq(
 Seq
```

Then, for $n = 6$, you enter the following:

```
seq(1/cos(X/3),X
,π/12,11*π/12,π/
6)→L₂
{1.003819838 1.…
```
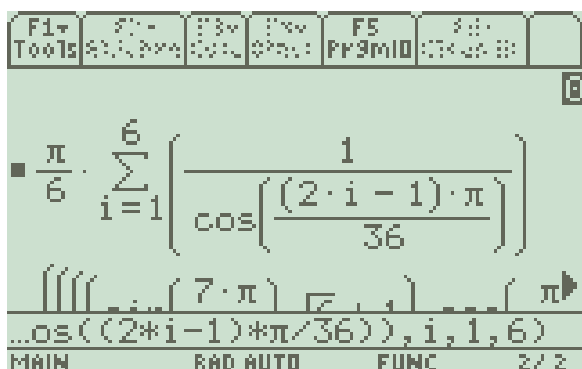
Which says to form the sequence of values of $\dfrac{1}{\cos(x)}$ as $x$ goes from the first midpoint of the first interval, $\dfrac{\pi}{12}$,

to the midpoint of the 6$^{th}$ interval, $\dfrac{11\pi}{12}$, by steps of size, $\dfrac{\pi}{6}$ and then store this sequence in $L_2$.

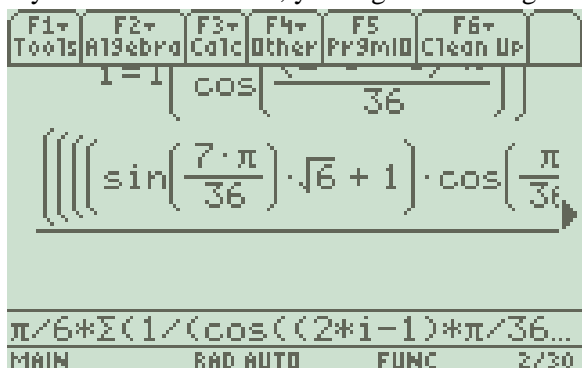All that remains is then to multiply the sum of these values by $\dfrac{\pi}{6}$:

```
seq(1/cos(X/3),X
,π/12,11*π/12,π/
6)→L₂
{1.003819838 1.…
π/6*sum(L₂)
         3.937938683
```

Note that you can also get the `seq()` and `sum()` functions from the `list` menu.

On the TI89 it's a bit simpler.  Enter $\dfrac{\pi}{6}$ and then find "sum" on the calculus menu and follow your nose.

```
F1▼    F2▼  F3▼ F4▼   F5        F6▼
Tools Algebra Calc Other Prgmio Clean Up
```

$$\blacksquare\ \frac{\pi}{6}\cdot\sum_{i=1}^{6}\left(\frac{1}{\cos\left(\dfrac{(2\cdot i - 1)\cdot\pi}{36}\right)}\right)$$

```
…os((2*i-1)*π/36)),i,1,6)
MAIN        RAD AUTO      FUNC      2/2
```

If you're in exact mode, you'll get something like this:

```
F1▼    F2▼  F3▼ F4▼   F5      F6▼
Tools Algebra Calc Other Prgmio Clean Up
```

$$\pi/6*\Sigma(1/(\cos((2*i-1)*\pi/36…$$
```
MAIN        RAD AUTO      FUNC      2/30
```

Which, if you arrow up and then right arrow, you can scroll through to see that it's

$$\frac{\left(\left(\left(\left(\left(\sin\left(\frac{7\pi}{36}\right)\cdot\sqrt{6}+1\right)\cdot\cos\left(\frac{\pi}{36}\right)+\sin\left(\frac{7\pi}{36}\right)\right)\cdot\cos\left(\frac{5\pi}{36}\right)+\sin\left(\frac{7\pi}{36}\right)\cdot\cos\left(\frac{\pi}{36}\right)\right)\cdot\cos\left(\frac{7\pi}{36}\right)+\sin\left(\frac{7\pi}{36}\right)\cdot\cos\left(\frac{\pi}{36}\right)\cdot\cdots\right.\right.}{6\cdot\sin\left(\frac{7\pi}{36}\right)\cdot\cos\left(\frac{\pi}{36}\right)\cdot\cos\left(\frac{5\pi}{36}\right)\cdot\cos\left(\frac{7\pi}{36}\right)}$$

Yikes!  So you want to be in approximate mode, or press the green diamond button before pressing `Enter`:

$$6 \cdot \sin\left(\frac{\pi}{36}\right) \cdot \cos\left(\ldots\right)$$

$$\blacksquare \frac{\pi}{6} \cdot \sum_{i=1}^{6}\left(\frac{1}{\cos\left(\frac{(2 \cdot i - 1) \cdot \pi}{36}\right)}\right)$$

$$3.93794$$

π/6*Σ(1/(cos((2*i−1)*π/36...

MAIN          RAD AUTO          FUNC          3/30

Which approximation agrees with the TI83.