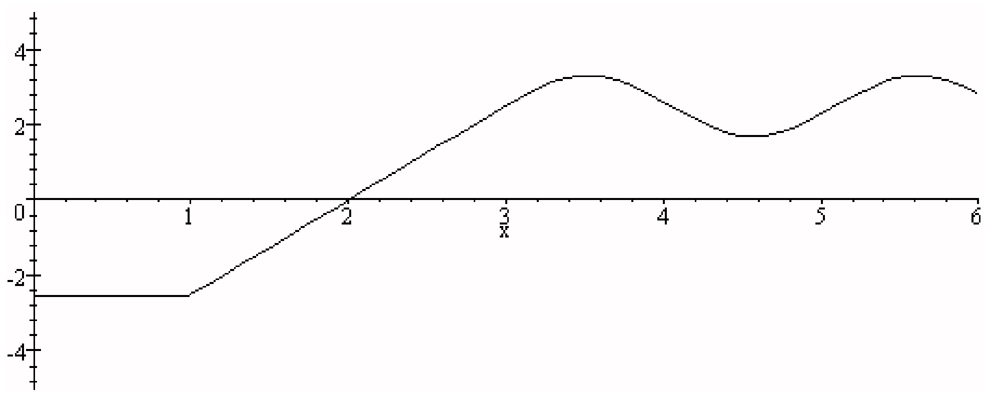
Math 1B -- Chapter 5 – A Sampling of Fair Game for Spring '10 Test

1. Find the approximating sum for $\int_0^2 \left(-16t^2+64\right)dt$ with $n=2$
   a. Assuming subintervals of equal length, find the value of Δt and t_i.
   b. Using right endpoints as sample points in a Riemann sum with subintervals of equal length.
   c. Using left endpoints as sample points in a Riemann sum with subintervals of equal length.
   d. Using midpoints as sample points in a Riemann sum with subintervals of equal length.
2. Use the definition of the definite integral to evaluate $\int_0^2 \left(-16t^2+64\right)dt$ as a limit of Riemann sums. Do not use the evaluation theorem. Recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$,

   $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

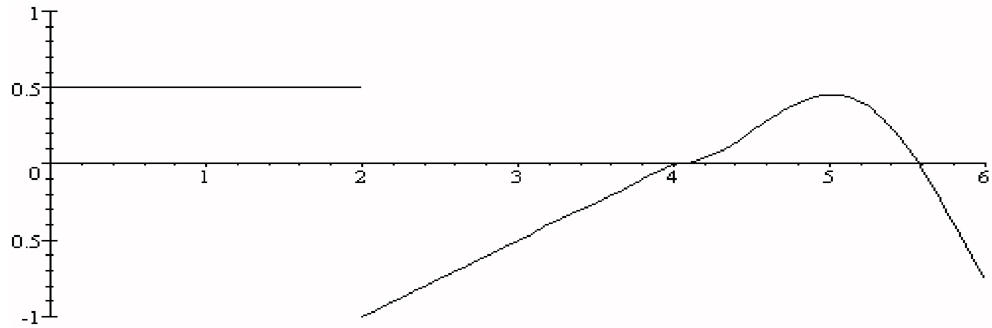
3. Evaluate the integral. Clearly indicate any substitution, or integration by parts.
   a. $\int_{-2}^{2} (5x+2)^3\, dx$
   b. $\int_0^{\pi/4} \sec\theta \tan\theta\, d\theta$
   c. $\int_0^1 x^2 \sin\left(x^3\right) dx$
   d. $\int_0^{\pi} x\sin(5x)\, dx$

4. Let $A(x)=\int_0^x f(t)\,dt$, where *f* is the function whose graph is shown below:



   a. Make a table of values showing the value of $A(x)$ at $x = 0, 1, 2, 3$ and estimating values of $A(x)$ at $x = 3.5, 4, 4.5, 5, 5.5$ and 6.
   b. Sketch a graph for $A(x)$ on $0 \le x \le 6$.
   c. What is the global minimum for $A(x)$? Estimate the global maximum.

5. Let $A(x)=\int_0^x f(t)\,dt$, where *f* is the function whose graph is shown below:



   d. Make a table of values showing the value of $A(x)$ at x = 0, 2, 3 and 4

   e. Sketch a graph for $A(x)$ on $0 \le x \le 6$.

   f. What are the global maximum and the global minimum for $A(x)$?

6. Consider the definite integral written as a limit of a Riemann sum:

$$\int_0^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x = \lim_{n\to\infty} \frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}\right)^2$$

   a. What is Δx here? What does that mean about the value of *b*?
   b. What is $f(x)$?
   c. What is c_k ?
   d. Compute the value of the limit.

7. Write a definite integral which is equivalent to this limit of a Riemann sum:

$$\lim_{n\to\infty} \frac{1}{2n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}$$

8. Simplify each expression (hint: none of these is difficult):

   a. $\int_0^{\pi/2} \frac{d}{dx}\sin^5 x\,dx$

   b. $\frac{d}{dt}\int_0^t \sin^5 x\,dx$

   c. $\frac{d}{du}\int_0^{\pi/2} \sin^5 u\,du$

   d. $\int_0^{\pi} \frac{d}{du} e^{u/\pi}\sin u\,du$

   e. $\frac{d}{dy}\int_0^{y} e^{y/\pi}\sin y\,dy$

   f. $\frac{d}{dx}\int_0^{\pi} e^{x/\pi}\sin x\,dx$

9. Use substitution to simplify the following definite integrals. For each, explicitly state what the components of your substitution are (on separate paper):

a. $\int_0^{\pi/4} \tan^3\theta \sec^2\theta \, d\theta$ $\qquad u = \tan\theta \quad du =$ ____________?

b. $\int_{-1}^{\theta} x^2 \cos\left(x^3+1\right) dx$, where $\theta = \sqrt[3]{\pi/2-1}$ $\qquad u =$ _____? $du =$ ___________?

10. $\int_0^{\pi/4} \frac{\tan^2\sqrt{x}}{\sqrt{x}} dx$ (*hint*: $\tan^2\theta + 1 = \sec^2\theta$) $\quad u =$ _____? $du =$ ___________?

11. Evaluate the definite integral $\int_0^e \frac{1}{5x\ln(5x)} dx$.

12. Evaluate the definite integral $\int_0^{\pi/15} \tan(5x) \, dx$. *hint:* $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

13. Consider the area of the region bounded by $y = x^2$ and $y = 2 - |x|$.
   a. Sketch a graph illustrating the region.
   b. By integrating over *x* and using symmetry, as appropriate.
   c. By integrating over *y* and splitting the region into 2 pieces, as appropriate.

14. Consider the area of the region bounded by $y = (x-1)^2$ and $y = 1 + \sin\left(\frac{\pi x}{2}\right)$.
   a. Sketch a graph illustrating the region.
   b. Compute the volume of revolution generated by revolving the region about the *x*-axis. Use either the shell method or the washer method, whichever seems easier.
   c. Compute the volume of revolution generated by revolving the region about the *y*-axis. Use either the shell method or the washer method, whichever seems easier.

15. Find the volume generated by revolving the area bounded by $y = 1 + \sin x$ and $y = \cos x$ for $0 \le x \le \pi$ about the line $y = -1$.

16. A hole of radius *r* is drilled through the center a sphere of radius $R > r$. Find the volume of the remaining portion of the sphere.