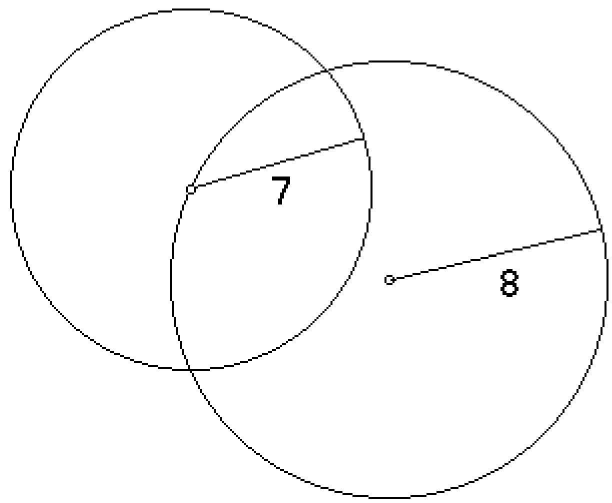
Math 1B - Fair Game for Test 2 – Spring '10 Name:___________________________
Write all responses on separate paper. Show your work for credit.

1. Consider the region bounded by two circles with radii 7 and 8, where the circumference of the larger circle passes through the center of the smaller circle, as shown:



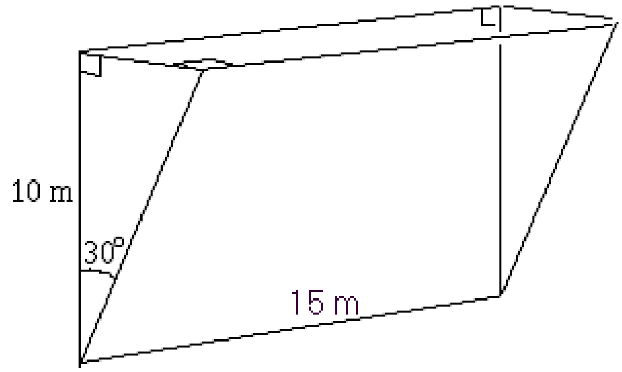
a. Introduce an appropriate coordinate system and sketch a graph showing how these circles are situated in your coordinate system.
b. Write equations for each circle relative to your coordinate system. These equations may be parametric, if you like.
c. Set up an integral to compute the area of this region in terms of one of your coordinate variables or the coordinate parameter. Don't simplify the integral.
d. Set up an integral to compute the volume generated by revolving this area around the line through the two circle centers using the 'shell' method.
e. Set up an integral to compute the volume generated by revolving this area around the line through the two circle centers using the 'washer' method.

2. What minimal energy is required to lift a 1000 kg mass to altitude of 3 km above the surface of the Earth? You will need to use the universal law of gravitation, which states that the gravitational attraction between two masses is given by $F = \frac{6.67259\times10^{-11}Mm}{r^2}$, where M and m are the two masses (in kilograms) and r is the distance between their centers. It may be useful to know that Earth's mass is approximately 5.97×10^{24} kilograms and the radius of Earth is approximately 6380 kilometers.

3. Two electrons r meters apart repel eachother with a force of $F = \frac{23\times10^{-29}}{r^2}$ newton . Suppose one electron is held fixed at the point (1,0) aon the x-axis (units in meters). How much work does it take to move a second electron along the x-axis form the point (-1,0) to the origin?

4. Find the volume generated by revolving the area in the first quadrant bounded by $y = 2x\sin(\pi x)$ and $y = 8x^3$ about the y-axis.

5. Suppose the trough shown below is filled with water. The 10 meter length is vertical. And the front face is tilted at a 30° angle, as shown:



Find the minimum total work required to empty the water out through the top of the trough.

6. Use trigonometric substitution to simplify $\int_2^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}\,dx$. State explicitly what your substitution is in each case…and remember to adjust the bounds of integration.

RSP: Let $x = 2\sec\theta$, then $\int_2^{2\sqrt{2}} \frac{x}{\sqrt{x^2-4}}\,dx = \int_0^{\pi/4} \frac{2\sec\theta}{2\tan\theta} 2\sec\theta\tan\theta\,d\theta = 2\tan\theta\Big|_0^{\pi/4} = 2$

7. A cylinder with horizontal axis is half full of water. Find, in terms of the cylinders radius *r* and length *h*, the work required to pump the water out of the cylinder through a pipe that extends one radius above the top of the cylinder.

8. Simplify $\int_0^1 \frac{x^2+2x-3}{(x-2)(x^2+1)}\,dx$. Note that the partial fractions form of the integrand is $\frac{A}{x-2}+\frac{Bx+c}{x^2+1}$, so your first task is to determine the values of *A*, *B* and *C* that work here.

RSP: $\frac{x^2+2x-3}{(x-2)(x^2+1)} = \frac{A}{x-2}+\frac{Bx+C}{x^2+1} \Leftrightarrow x^2+2x-3 = A(x^2+1)+(Bx+C)(x-2)$

$\Leftrightarrow \begin{matrix} A+B=1 \\ -2B+C=2 \\ A-2C=-3 \end{matrix} \Rightarrow \begin{matrix} B+2C=4 \\ -2B+C=2 \end{matrix} \Rightarrow 5C=10 \Rightarrow B=0 \Rightarrow A=1$. Thus

$$\int_0^1 \frac{x^2+2x-3}{(x-2)(x^2+1)}\,dx = \int_0^1 \frac{1}{x-2}+\frac{2}{x^2+1}\,dx = \ln|x-2| + 2\arctan x\Big|_0^1 = \frac{\pi}{2}-\ln 2$$

9. Use substitution and integration by parts to simplify $\int_0^1 x^3 e^{x^2}\,dx$

10. Find an elementary antiderivative for $\int \sqrt{4+x^2}\,dx$.

11. Evaluate $\int_0^\pi e^{-3x}\cos(x)\,dx$. *Hint:* Use integration by parts to establish a recursion formula.

12. Rewrite the integral $\int_0^\pi \cos(2x)e^{-\sin(2x)}dx$ using the substitution $u=\sin(2x)$. What is *du*? What are the new bounds of integration? Evaluate the integral.

13. Use substitution to simplify the following definite integrals. For each, explicitly state what the components of your substitution are (on separate paper):

a. $\int_0^{\pi/4} \tan^3\theta \sec^2\theta \, d\theta$ $\quad u = \tan\theta \quad du =$ ____________?

b. $\int_{-1}^{\theta} x^2 \cos\left(x^3+1\right) dx$, where $\theta = \sqrt[3]{\pi/2-1}$ $\quad u =$ _____? $\quad du =$ ___________?

c. $\int_0^{\pi/4} \frac{\tan^2\sqrt{x}}{\sqrt{x}} dx$ ($hint$: $\tan^2\theta + 1 = \sec^2\theta$) $\quad u =$ _____? $\quad du =$ ___________ ?

14. Use integration by parts to compute the following definite integrals. For each, explicitly state what the components of your substitutions are (on separate paper):

d. $\int_0^1 x^2 e^{-2x} dx$ $\quad u =$ ___________ $\quad dv =$ ___________ $\quad du =$ ___________ $\quad v =$ ___________

e. $\int_0^1 x\sqrt{1-x}\, dx$ $\quad u =$ ____x____ $\quad dv =$ ___________ $\quad du =$ ___________ $\quad v =$ ___________

15. Use a trigonometric substitution to evaluate $\int_0^1 x\sqrt{4-x^2}\, dx$.

16. Let $I = \int_0^{2\pi} e^{-2x} \sin 4x \, dx$. Do integration by parts twice so that I recurs on the right side of your equation. Solve the equation for *I*.

17. Let $I_n = \int_0^{\pi/2} x^n \sin x \, dx$. Prove that $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$.

18. Use the Fundamental Theorem of calculus to find $f(8)$ if $x^2 \arctan\left(\frac{x}{2}\right) = \int_0^{x^3} f(t)\, dt$

19. Show how to use the methods of partial fractions and completing the square to compute $\int_{-3}^{-2} \frac{9x+25}{(x-1)\left(x^2+6x+10\right)} dx = \ln\left(\frac{9}{32}\right) + \frac{\pi}{4}$. *Hint: the antiderivative may involve two logarithms and an arctan function.*

20. The Gamma Function is defined by $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$

a. Find $\Gamma(1)$

b. Find $\Gamma(2)$